Seminar in DMGW Group

Photon propagation in neutron stars magnetosphere and Dirac materials

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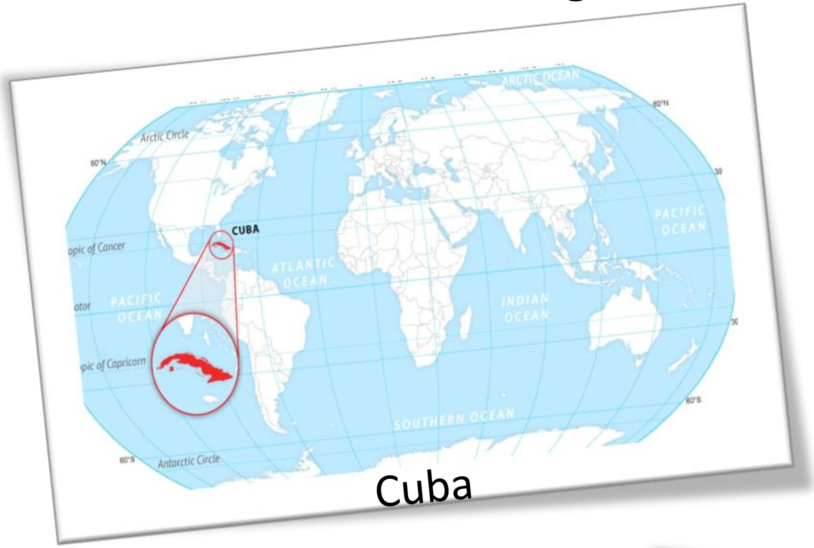
ICIMAF

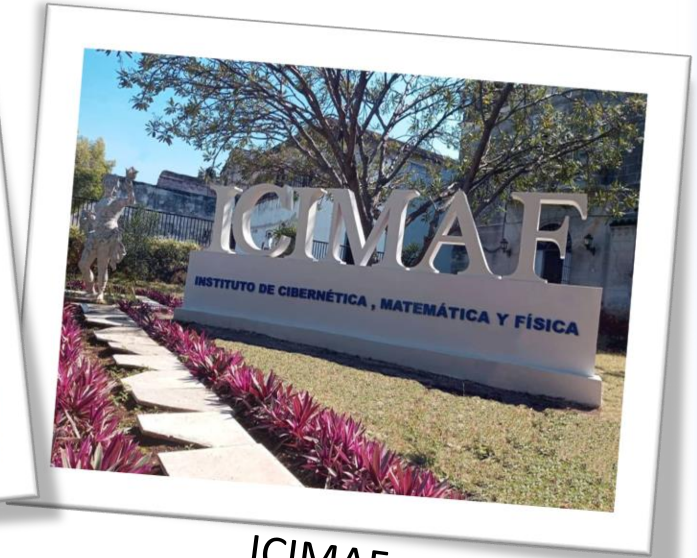
Who I am?

Adrian William Romero Jorge



Cuba

University of Havana



ICIMAF

Theoretical Physics

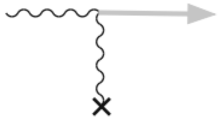
Motivation

- Photon propagation with an external magnetic field

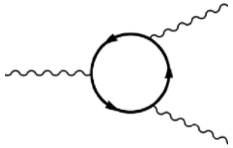
1,2,3,4,5,6,7,8,9,10

Campo Magnético Crítico $B_c = \frac{m_e^2 c^2}{e\hbar} = 4{,}4 \times 10^{13}\ \text{G}$ $E_c = \frac{m_e^2 c^3}{e\hbar} = 1{,}3 \times 10^{18}\ \text{V/m}$

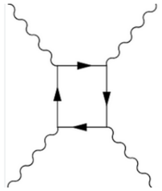
Vacuum Dichroism



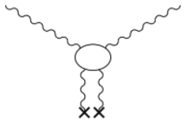
Photon Splitting



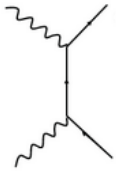
Photon-photon Scattering



Vacuum Birefringence



Pair Creation



1- J. Schwinger, 1951
2- Johnson and Lippman, 1949
3- B. Shabad, 1972
4- H. P. Rojas, 1975
5- S. Masood, 2020
6- V. Katkov, 2016
7- Euler and Kockel, 1935.
8- Mignani, 2016
9- Rizzo, 2014
10- Rojas, 2018

Motivation

1,2,3,4,5,6,7,8,9,10

- Photon propagation with an external magnetic field

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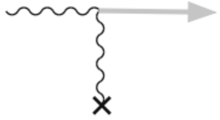

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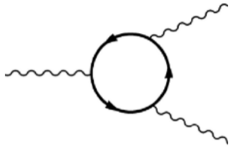
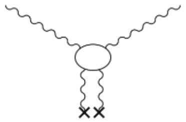


Photon-photon Scattering

Vacuum Birefringence



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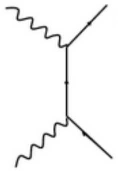

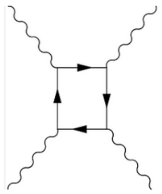

- Relevance

Quantum Hall Effect | Faraday Effect | Cotton-Mouton Effect

- Heavy Ions Collisions
- Astrophysics and Cosmology
- Physics of Materials
- Condensed Matter
- Comunications

- Experiments to detect vacuum effects (birefringence)

1- *J. Schwinger, 1951*
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10- *Rojas, 2018*

Experiments

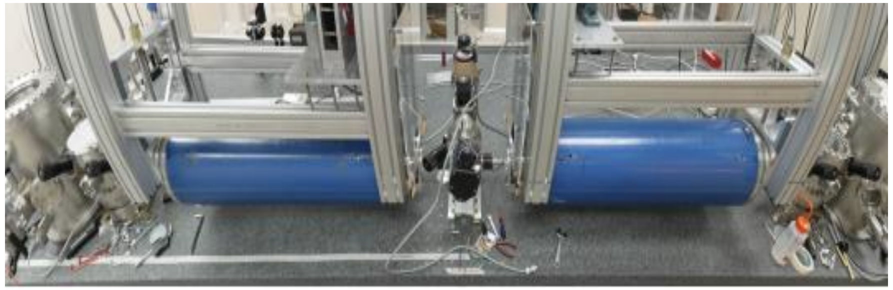
$$B_c = 4.41 \times 10^{13}\ G$$

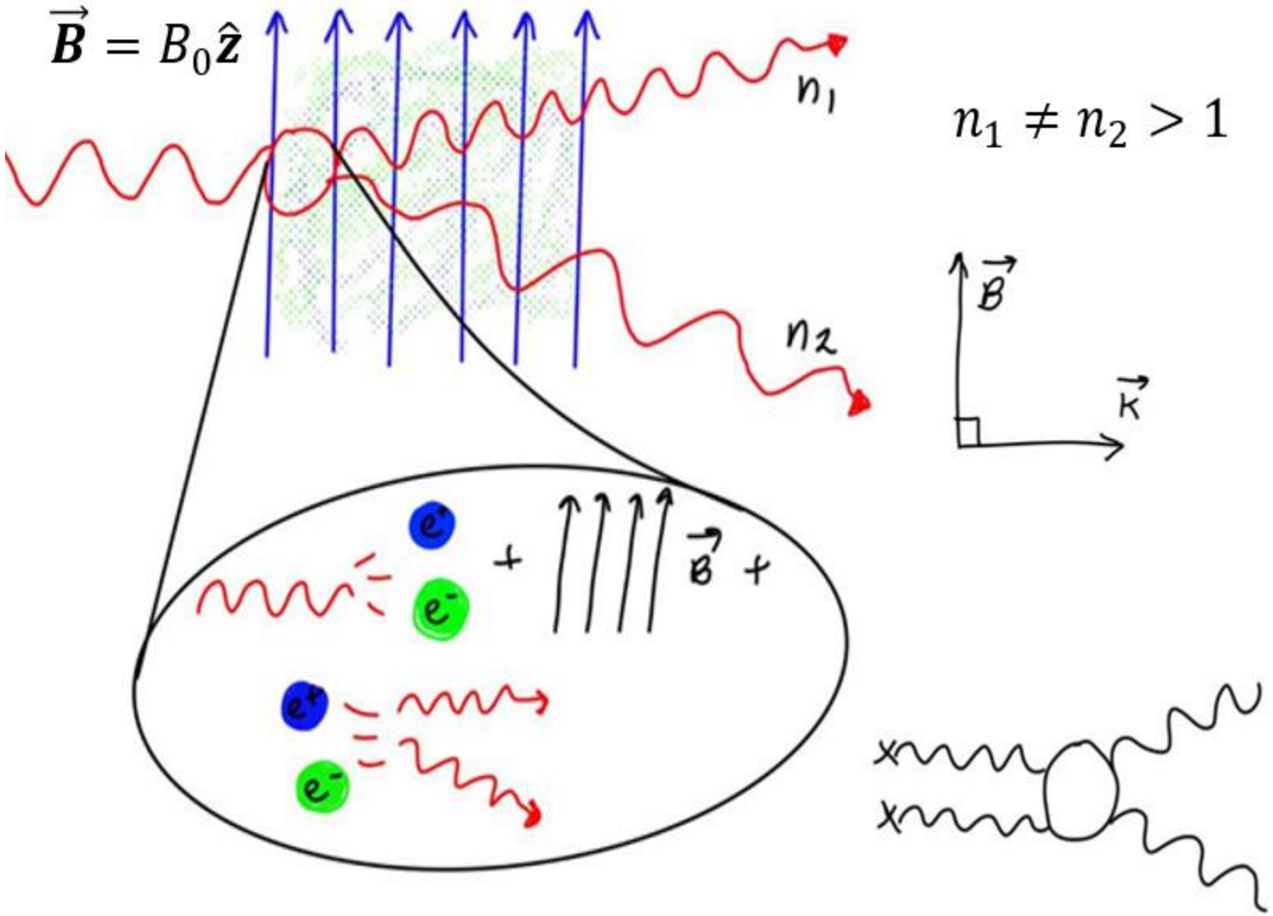
Generate intense magnetic fields

Birefringence

- PVLAS Experiment [a]
- OVAL Experiment [b]
- Others [c]

$$B \sim 10^4\ G \qquad I \sim 10^{15}\ W/m^3$$





[a] *Ejlli, 2020.* [b] *Xing Fan, 2017.* [c] *Karbstein, 2021.*

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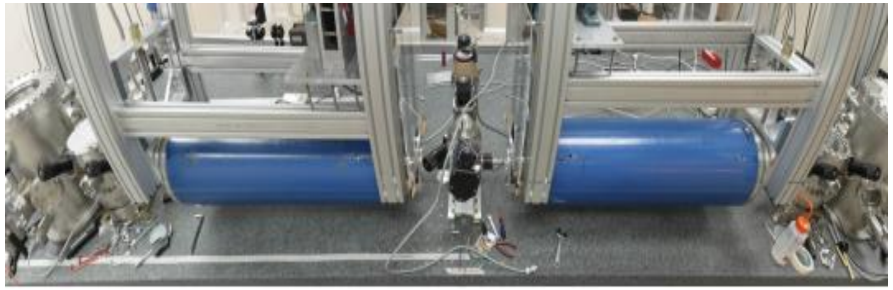

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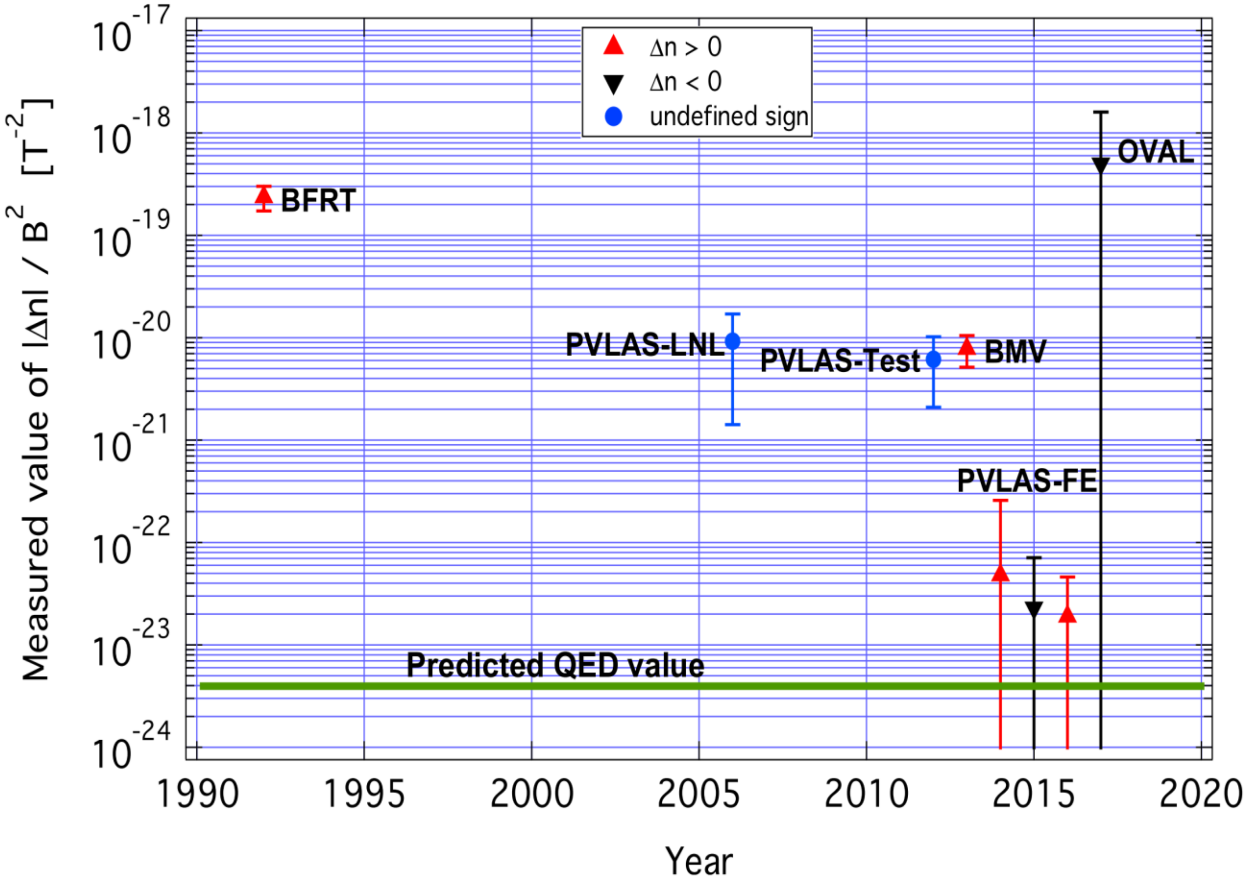

$$\Delta n_{the}/B^2 \sim 10^{-24} \qquad \Delta n_{exp}/B^2 \sim 10^{-20}$$

$$Physics\ beyond\ the\ SM, axions$$

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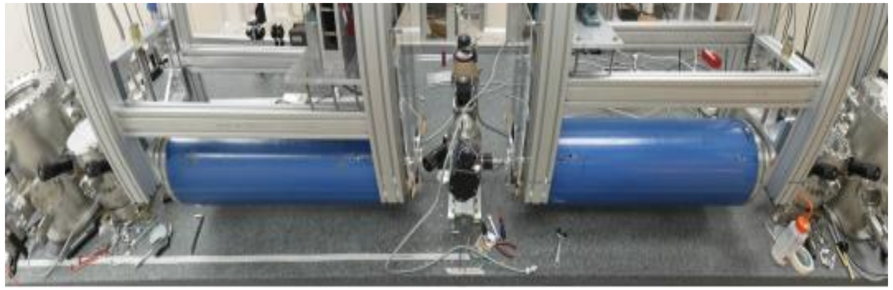
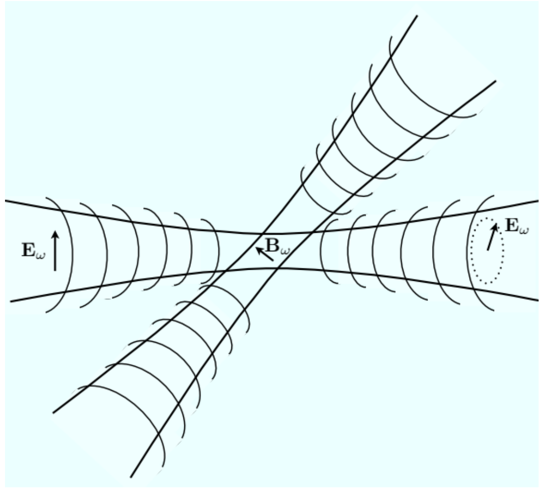


[a] *Ejlli, 2020.* [b] *Xing Fan, 2017.* [c] *Karbstein, 2021.*

Use lasers to induce the photon dispersion

- BMV Experiment [d]
- HERCULES Experiment [e]
- Others [c]



$I \sim 10^{26}\ W/m^3$

$B \sim 10^{10}\ G$

small Δt

[d] *Rizzo, 2014.* [e] *Tommasini, 2009.*

Astrophysical Observations

$$B_c = 4.41 \times 10^{13}\ G$$

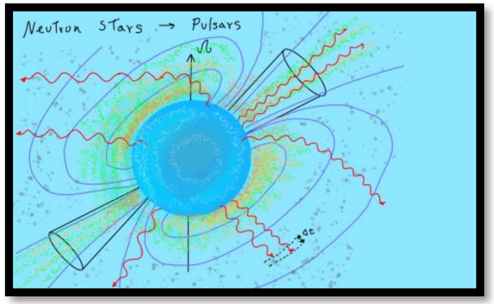
Measure photon time delay

- BeppoSAX satellite, GRBs [f]
- EGRET and GLAST Detectors [g]
- OLFAR Antennas $\Delta\tau \sim ms$ [h]
- Rossi X-ray Timing Explorer $\Delta\tau \sim \mu s$ [i]
- Ooty radio telescope at RAC $\Delta\tau \sim ms$ [j]

$\Delta\tau_V \sim ns$ *Denisov, 2017.*

$\Delta\tau_M \sim ms$

$\delta\tau \sim 10 - 100\ ns$ *Bailes, 2009.*



[f] *Kumar, 1994.*
[g] *Katz, 1994 y Wang, 2004.*
[h] *Bentum, 2016.*
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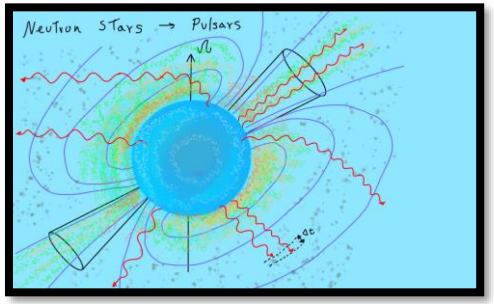

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Evidence of Birefringence

- Very Large Telescope RX J1856.5−4 [l]

Photon Dispersion in HIC (Experiment)

- Au-Au in RHIC [k]

$\sqrt{s_{NN}} = 200\ GeV$

$B \sim 10^{18}\ G$

[k] Y. *Zhong, 2014.*
[l] Mignani, *2016.*

Motivation

Objetive

Study photon propagation in the presence of an external magnetic field for two physical scenarios: astrophysics (pulsars) and condensed matter (Dirac materials).

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Photon Propagation

$\mathbf{k} \parallel \mathbf{B}$

- Vacuum
- Medium

H. P. Rojas, 2010
Cruz Rodríguez, 2016
Romero Jorge, 2021

Faraday

Hall

$\mathbf{k} \perp \mathbf{B}$

- Vacuum
- Medium

H. P. Rojas, 2014
Romero Jorge, 2020

Birefringence

Time Delay

Motivation

Objetive

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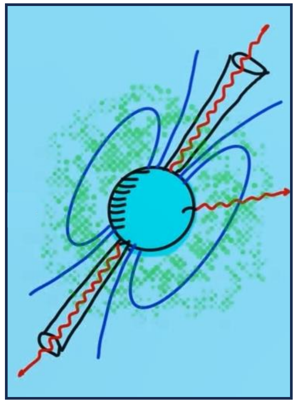
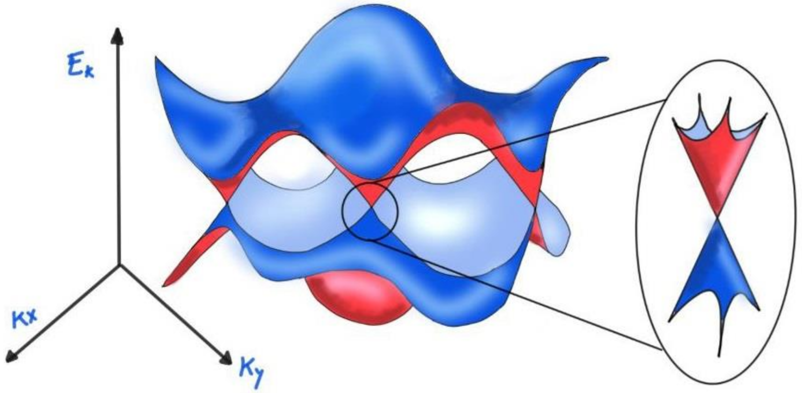
Birefringence

Time Delay

Aplications





Methodology

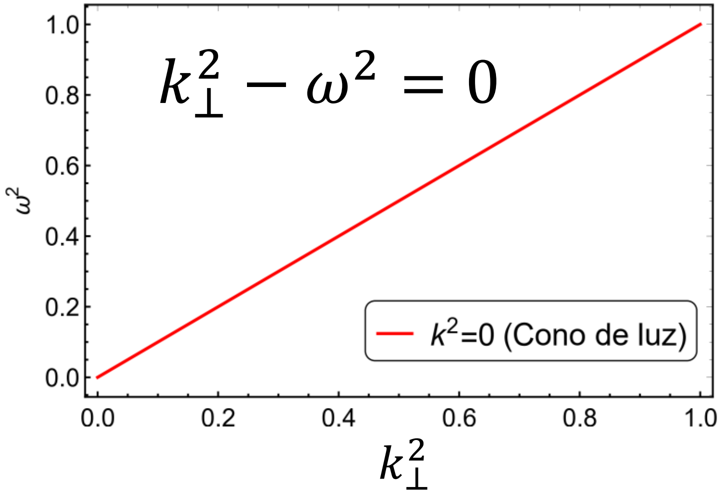
Natural Units

$\hbar = c = 1$

Propagation of light in a vacuum: Maxwell's Classical Theory

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \Rightarrow k^2 a^\mu(x) = 0, \quad \partial_\nu F^{\nu\mu} = 0$$

$$F_{\mu\nu} = \partial_\nu A_\rho - \partial_\rho A_\nu \qquad \mu, \nu = 0,1,2,3$$



$F_{\mu\nu}$ → Electromagnetic field tensor

A_μ → Cuatri-potential vector

$\bar{\psi}, \psi$ → Cuatri-spinors field

γ^μ →Gamma matrices

e, m_e → Electron charge and mass

$\Pi_{\mu\nu}$ → Polarization Tensor

κ → Eigenvalues of the Polarization Tensor

a_μ → Cuatri-potential vector of the wave

A_μ^{ext} → External Cuatri-potential vector

Propagation of light in the magnetized vacuum of QED

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m_e]\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - ie\bar{\psi}\gamma_\mu A_\mu \psi + \mathcal{L}^{EH}$$

NLED

$$\Pi_{\rho\nu} a_\nu^{(i)} = \kappa_V^{(i)} a_\rho^{(i)}$$

Romero Jorge, Bsc. Thesis, 2021

$$A_\mu = A_\mu^{ext} + a_\mu$$

$$k_\perp^2 - \omega^2 = \kappa_V^{(i)}(\omega, k_\perp, B)$$

Euler-Heisenberg Lagrangian $\mathcal{L}^{EH}$

Euler and Kockel, 1935.

Methodology

Propagation of light in the magnetized vacuum of QFT

Natural Units

$\hbar = c = 1$

$$\mathcal{L}^{EH} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - ie\bar{\psi}\gamma_\mu A_\mu \psi - \frac{1}{8\pi^2}\int_0^{i\infty}\frac{ds}{s^3}e^{-m_e^2 s}\left[(es)^2\tilde{a}\tilde{b}\coth(e\tilde{a}s)\cot(e\tilde{b}s) - \frac{(es)^2}{3}\left(\tilde{a}^2 - \tilde{b}^2\right) - 1\right]$$

$$\tilde{a} = \left[\left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} + \mathcal{F}\right]^{1/2}$$

$$\tilde{b} = \left[\left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} - \mathcal{F}\right]^{1/2}$$

$$\mathcal{F} = \frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{2}\left(-\epsilon_0 E_T^2 + \frac{B_T^2}{\mu_0}\right), \quad (\boldsymbol{E_e} = \boldsymbol{0})$$

$$\mathcal{G} = \frac{1}{4}F^{\mu\nu}\tilde{F_{\mu\nu}} = \sqrt{\frac{\epsilon_0}{\mu_0}}(-\boldsymbol{E_T}\cdot\boldsymbol{B_T})$$

$$\mathcal{F} = \frac{B_\omega^2}{2} + \frac{B^2}{2} + \boldsymbol{B_\omega}\cdot\boldsymbol{B} - \frac{E_\omega^2}{2}$$

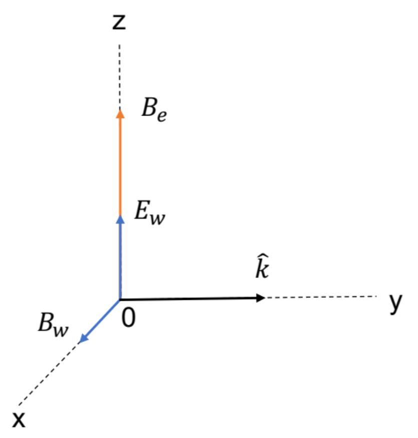
$$\mathcal{G} = -\boldsymbol{E_\omega}\cdot\boldsymbol{B}$$

$\mathcal{F}$ $\mathcal{G}$ → Gauge and Lorentz Invariants of the Electromagnetic field

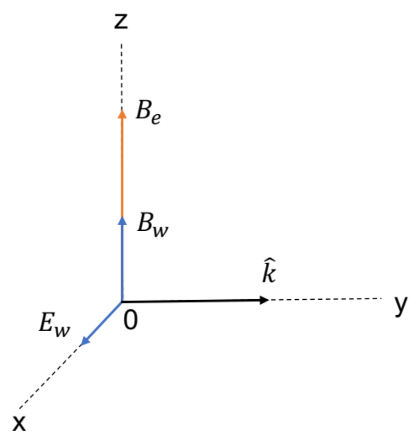
E_ω, B_ω → Electric/Magnetic Field of the wave

E, B y E_t, B_t → Total and External Electric/Magnetic respectively

Mode 2



Mode 3



Methodology

Propagation of light in the magnetized vacuum of QFT

Natural Units

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$$\tilde{a} = \left[\left(\mathcal{F}^2 + \mathcal{G}^2\right)^{1/2} + \mathcal{F}\right]^{1/2}$$

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$$\mathcal{F} = \frac{B_\omega^2}{2} + \frac{B^2}{2} + \boldsymbol{B}_\omega \cdot \boldsymbol{B} - \frac{E_\omega^2}{2} \qquad \mathcal{G} = -\boldsymbol{E}_\omega \cdot \boldsymbol{B}$$

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$\omega \leq 2\, m_e$

$B_\omega, E_\omega \ll B$

$$\mathcal{L}^{EH} = \frac{(1-\mathcal{L}_\mathcal{F})}{2}(E_\omega^2 - B_\omega^2) + (1-\mathcal{L}_\mathcal{F})(\boldsymbol{B}\cdot\boldsymbol{B}_\omega) + \frac{\mathcal{L}_{\mathcal{F}\mathcal{F}}}{2}(\boldsymbol{B}\cdot\boldsymbol{B}_\omega)^2 + \frac{\mathcal{L}_{GG}}{2}(\boldsymbol{B}\cdot\boldsymbol{E}_\omega)^2$$

Dispersion Equation

$$\kappa_V^{(2)} = \mathcal{L}_{GG}\mathrm{B}^2\,\mathrm{k}_\perp^2$$

$$\kappa_V^{(3)} = \mathcal{L}_{\mathcal{F}\mathcal{F}}\mathrm{B}^2\,\mathrm{k}_\perp^2$$

Phase Velocity

$$v_V^{(2)} = (1 - \mathcal{L}_{\mathcal{F}\mathcal{F}}(B)\,B^2)^{1/2}$$

$$v_V^{(3)} = (1 - \mathcal{L}_{GG}(B)B^2)^{1/2}$$

$\mu, \epsilon,$
M, P

$$\mathcal{L}_{EH}^{WF} = -\mathcal{F} + \frac{\xi}{4}(4\mathcal{F}^2 + 7\mathcal{G}^2),$$

$$\xi = \frac{8\alpha^2\hbar^3}{45m_e^4c^5} \sim \frac{8\alpha}{45B_c^2}$$

Methodology

Propagation of light in the magnetized medium

Natural Units

$\hbar = c = 1$

$\partial_\nu F_{\nu\rho} + \Pi_{\rho\nu} a_\nu = 0$ $[k^2 g_{\mu\nu} + \Pi_{\mu\nu}] a^\nu = 0$

$$\Pi_{\rho\nu} a_\nu^{(i)} = \kappa_M^{(i)} \, a_\rho^{(i)}$$

$$k_\perp^2 - \omega^2 = \kappa_M^{(i)}(\omega, k_\perp, B, \mu, T)$$

$$G(x, x' | A^{ext})$$

$$A_\mu^{ext} = B x_1$$

$$n_e, n_p$$

$$\varepsilon = \sqrt{p_3^2 + m_e^2 + 2eBn_l}$$

n_e, n_p → FD Distributions for e^- y e^+

$g_{\mu\nu}$ → Metric Tensor: Minkowski

$\Pi_{\mu\nu}$ → Polarization Tensor

κ → Eigenvalues of the Polarization Tensor

μ, T → Chemical Potential and Temperature

A_μ^{ext} → External Cuatri-potential vector

a_μ → Cuatri-potential vector of the wave

p_3 → Momentum in x_3 of the fermios

n_l → Landau Levels

Methodology

Propagation of light in the magnetized medium

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Transversal Modes

$$\kappa_M^{(2,3)} = \frac{q+t}{2} \pm \frac{1}{2}\sqrt{(q-t)^2 - 4r^2}$$

Longitudinal Mode

$$\kappa_M^{(1)} = t$$

Jorge Acosta & H. P. Rojas, 2023.

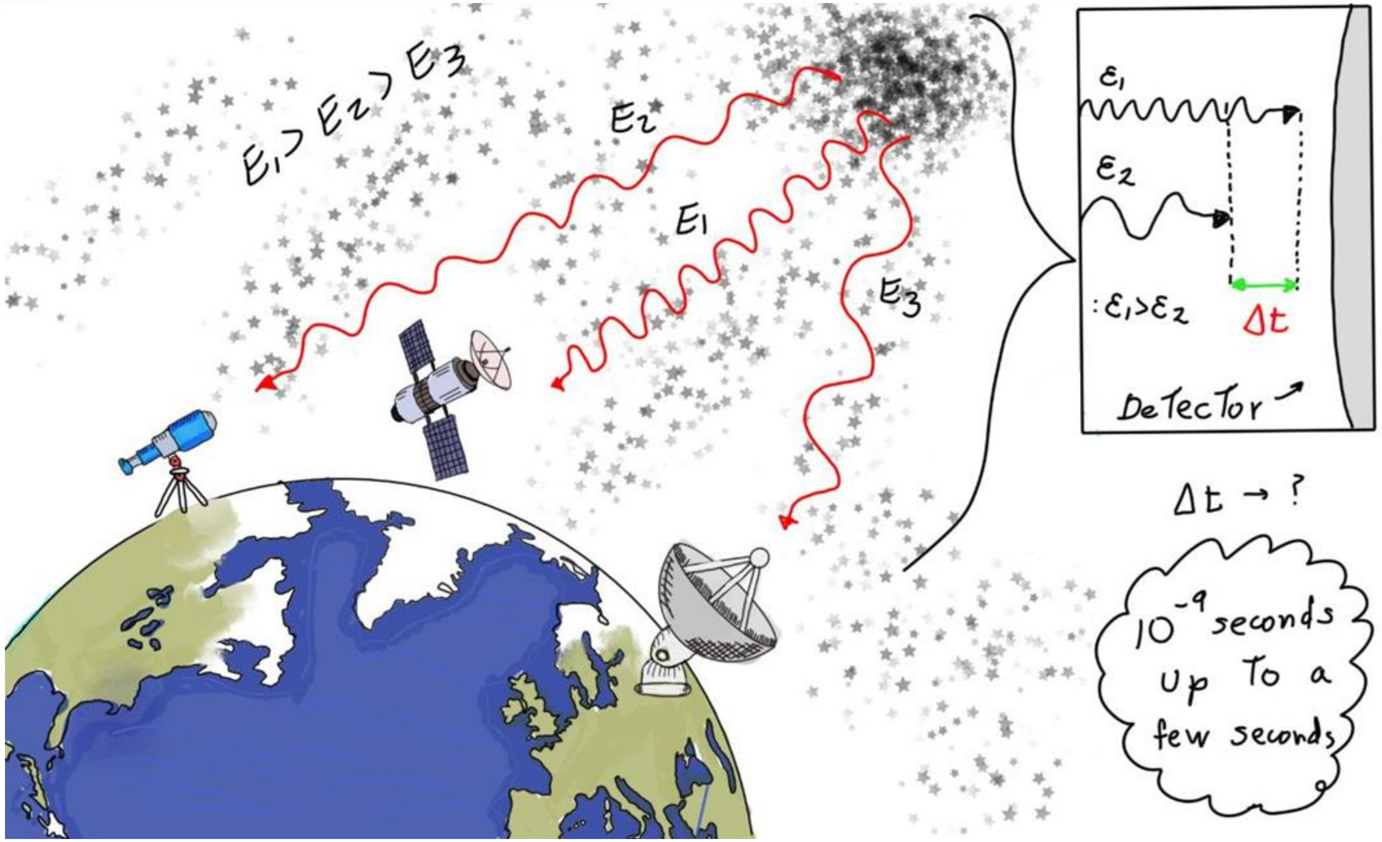
- Degenerated Gas limit $(T \ll \mu)$

$$n_L = \frac{\mu^2 - m_e^2}{2eB}$$

$$\kappa^{(i)} = \kappa_M^{(i)}(\omega, k_\perp, B, \mu, T)$$

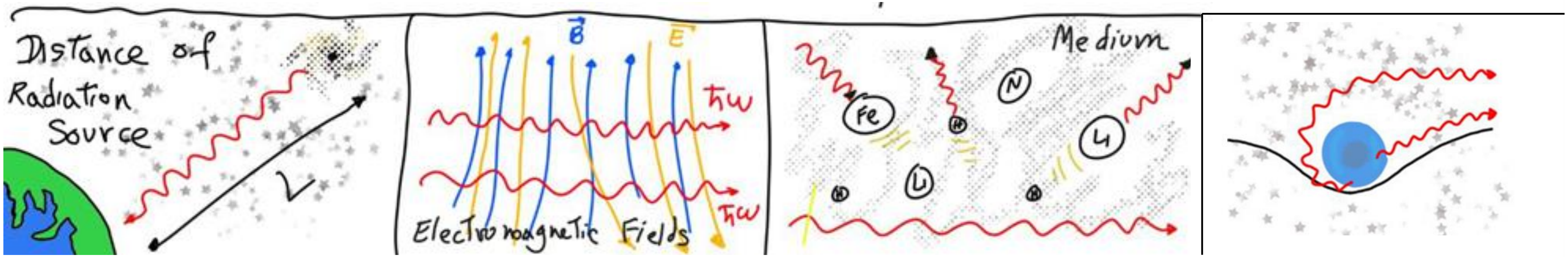
$$v_f^M(\omega, B, \mu) = \frac{\omega}{k_\perp(\omega, B, \mu)}$$

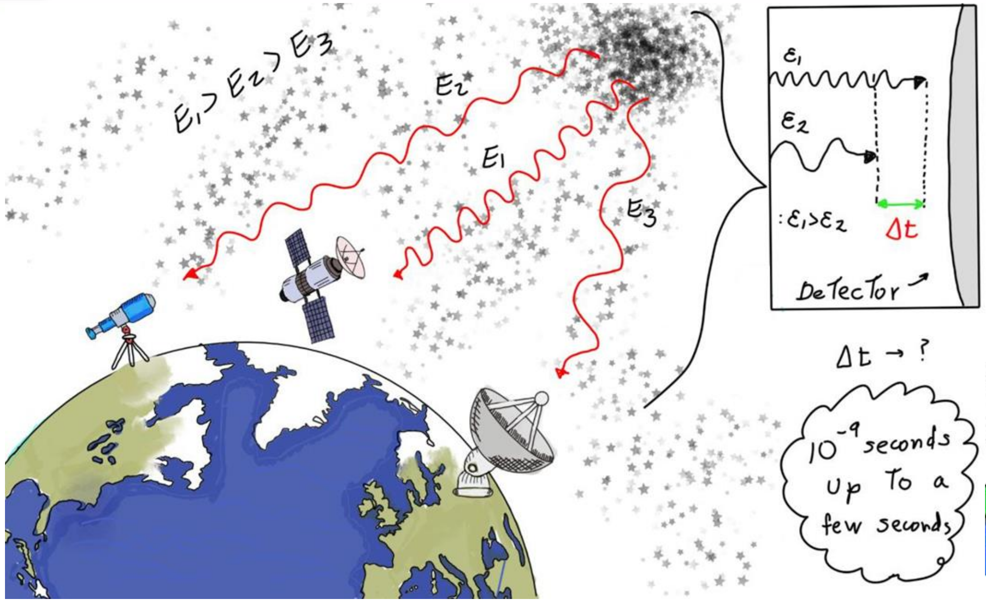
Photon Dispersion in Pulsars



Sources

- Cosmic Rays
- Neutron Stars
- Black Holes
- Galaxies





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Galaxies

Black Holes

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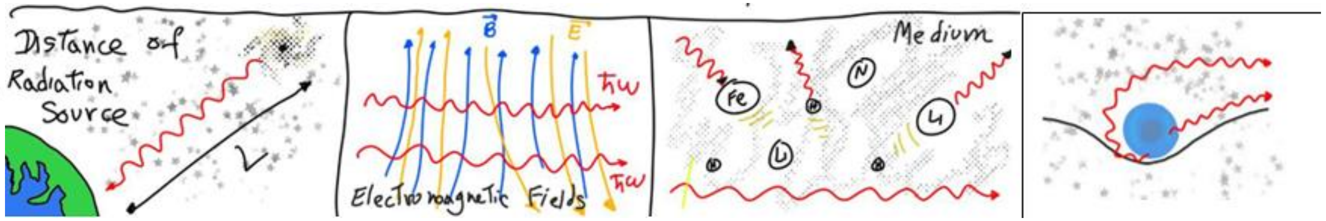
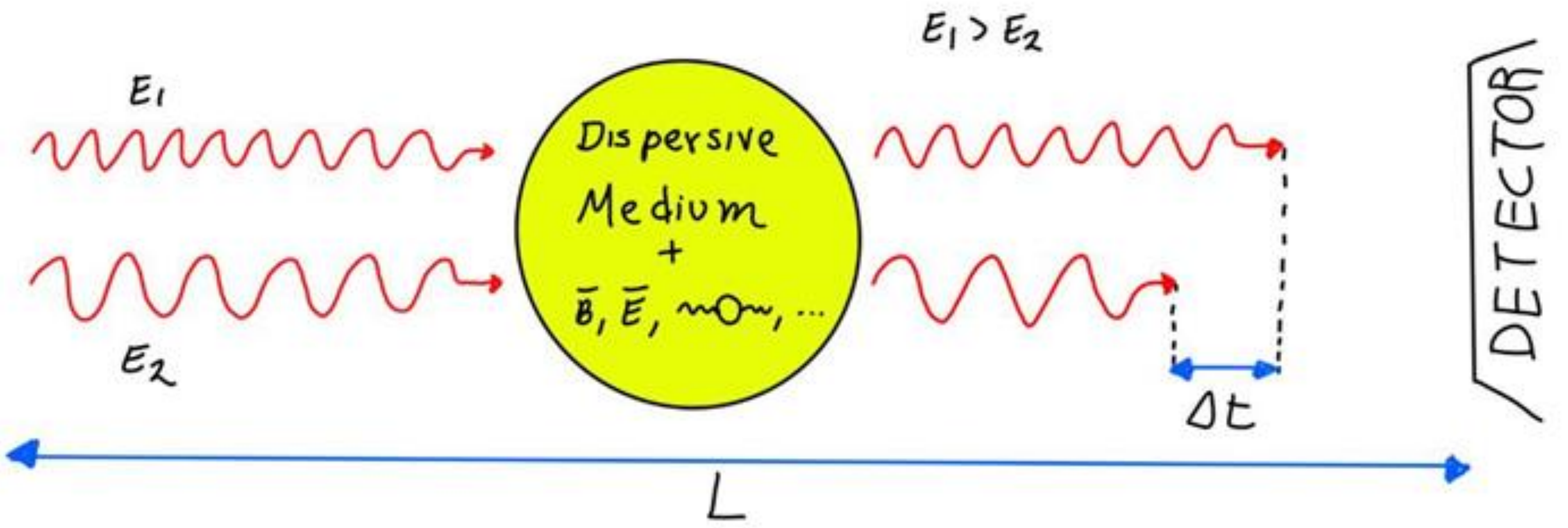
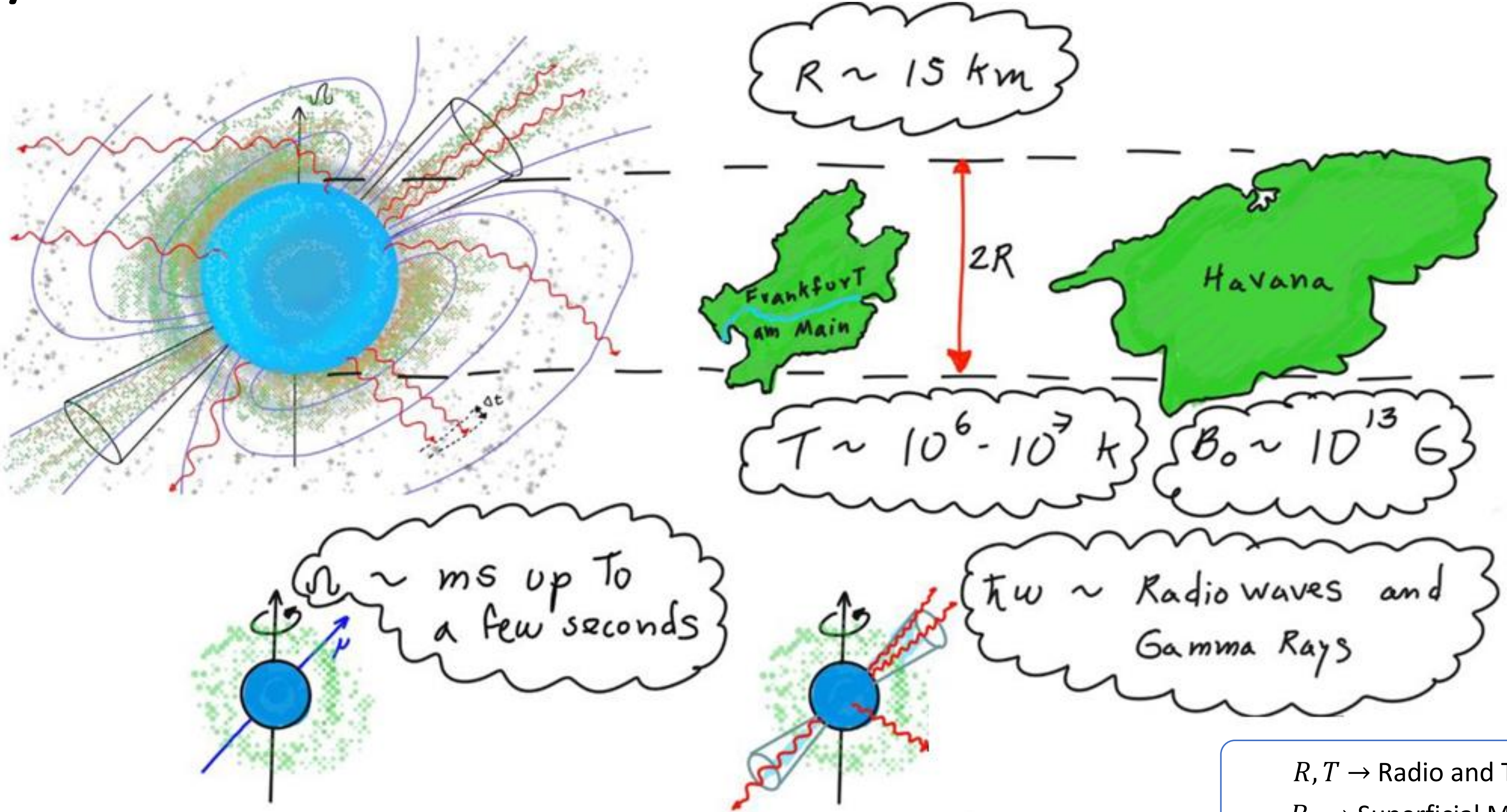




$$\Delta t = \left| t_{E_1} - t_{E_2} \right|$$

Physical Scenario: Neutron Stars



$R, T \rightarrow$ Radio and Temperature

$B_0 \rightarrow$ Superficial Magnetic field

$\Omega, \omega \rightarrow$ Angular Velocity and frequency

Contributions to time delay

$$\Delta\tau_{total} \rightarrow \Delta\tau_B, \ \Delta\tau_D, \ \Delta\tau_M, \ \Delta\tau_G$$

$\Delta\tau_{total}$ → Total Time delay

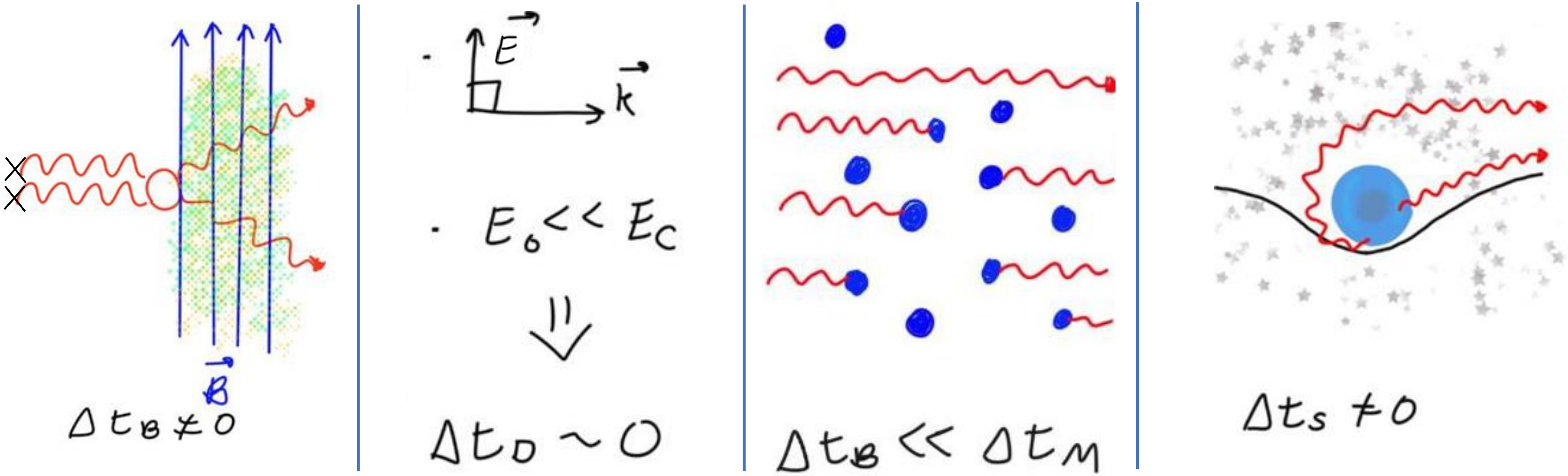
$\Delta\tau_B$ → Time delay by Birefringence

$\Delta\tau_D$ → Time delay by Dichroism

$\Delta\tau_M$ → Time delay by Medium

$\Delta\tau_G$ → Time delay by Space-Time Curvature

E_c, E_0 → Critical and Superficial Electric Field



Contributions to time delay

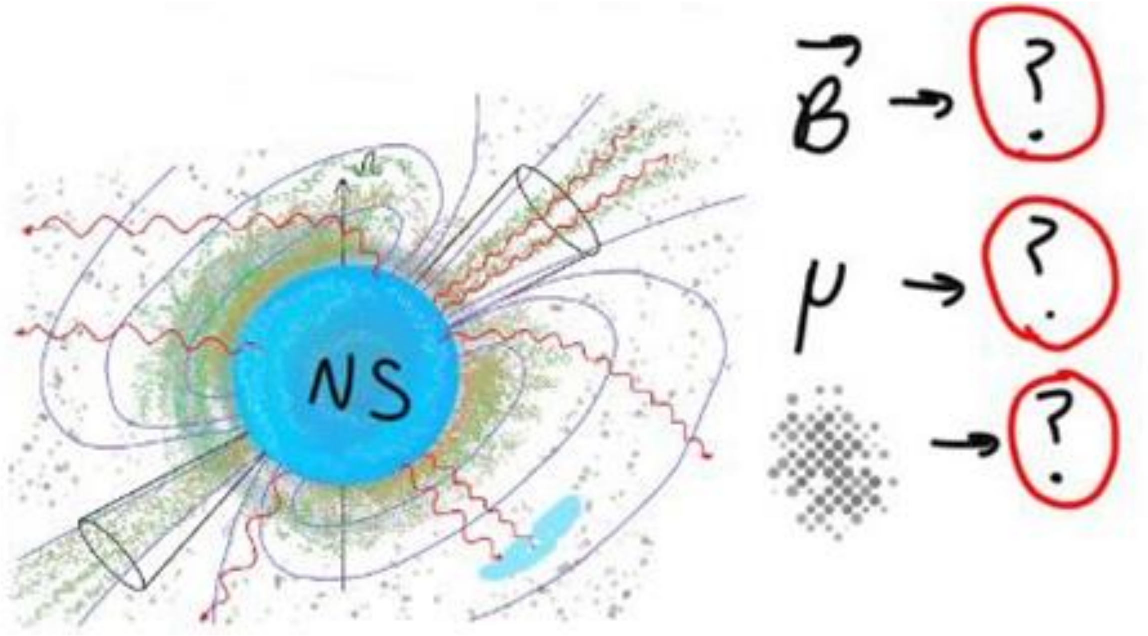
B → External magnetic field

μ → Chemical potantial

$$\Delta\tau_{total} \rightarrow \Delta\tau_B,\ \ \Delta\tau_M,\ \ \Delta\tau_G$$

Vacío → $B(r,t)$

Medio → $B(r,t)$, $\mu(r,t)$



Our Magnetosphere Model

Magnetized Medium

- Toroidal Magnetic Field

$$\boldsymbol{B}(\boldsymbol{r},t) = B_0 \frac{r_0}{r} \tanh(\Psi(\theta,\varphi,\chi,\Omega,t)\,)\,\hat{\varphi} \quad [m]$$

- Variable Chemical Potential

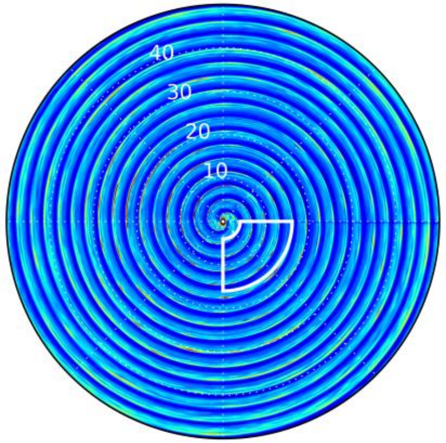
$$\mu(\boldsymbol{r},t) = \frac{(\mu_{max}-\mu_{min})\text{sech}^2\Psi+\mu_{min}}{\tanh\Psi}\frac{r_0}{r} + \mu_0 \quad [n]$$

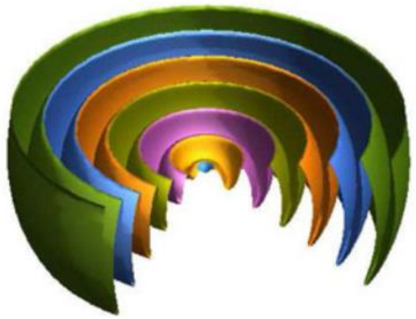
- Star with rotation effects

$$\Psi = \cos\theta\, cos\chi + sin\theta\, sin\chi \cos(\varphi - \Omega t) \qquad t = t_0 = 0$$

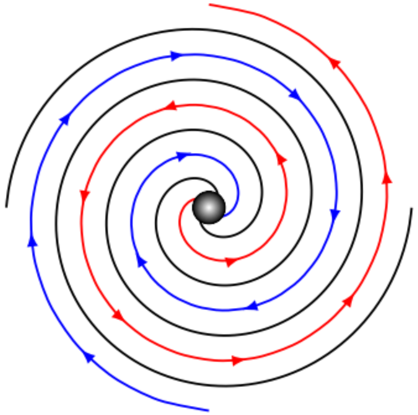
- Magnetosphere of an $e^- - e^+$ gas
- Space-Time Curvature

$$ds^2 = \left(1-\frac{2M}{r}\right)dt^2 - \left(1-\frac{2M}{r}\right)^{-1} dr^2 - r^2(d^2\theta + \sin^2\theta d^2\phi)$$





Pétri, 2016.



Magnetic Field lines, like a Parker Spiral

μ_{max}, μ_{min} → Max. and min. Chemical Potential
μ_0 → Chemical Potential to far distance
M, r → Mass and radio of the star
χ → Angle between $\boldsymbol{\Omega}$ and magnetic moment of the star
θ, φ → Azimuth and polar angle
Ω → Angular velocity of the star

[m] *Bogovalov, 1999. Michel, 1991. Corotini 1990.* [n] *Pétri, 2011, 2016, 2018.*

Our Magnetosphere Model

Magnetized Medium

- Toroidal Magnetic Field

$$\boldsymbol{B}(\boldsymbol{r},t) = B_0 \frac{r_0}{r} \tanh(\Psi(\theta,\varphi,\chi,\Omega,t))\,\hat{\varphi} \quad \text{[m]}$$

- Variable Chemical Potential

$$\mu(\boldsymbol{r},t) = \frac{(\mu_{max}-\mu_{min})\text{sech}^2\Psi+\mu_{min}}{\tanh\Psi}\frac{r_0}{r} + \mu_0 \quad \text{[n]}$$

- Star with rotation effects

$$\Psi = \cos\theta\,\cos\chi + \sin\theta\,\sin\chi\,\cos(\varphi - \Omega t) \qquad t = t_0 = 0$$

- Magnetosphere of an $e^- - e^+$ gas
- Space-Time Curvature

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d^2\theta + \sin^2\theta d^2\phi)$$

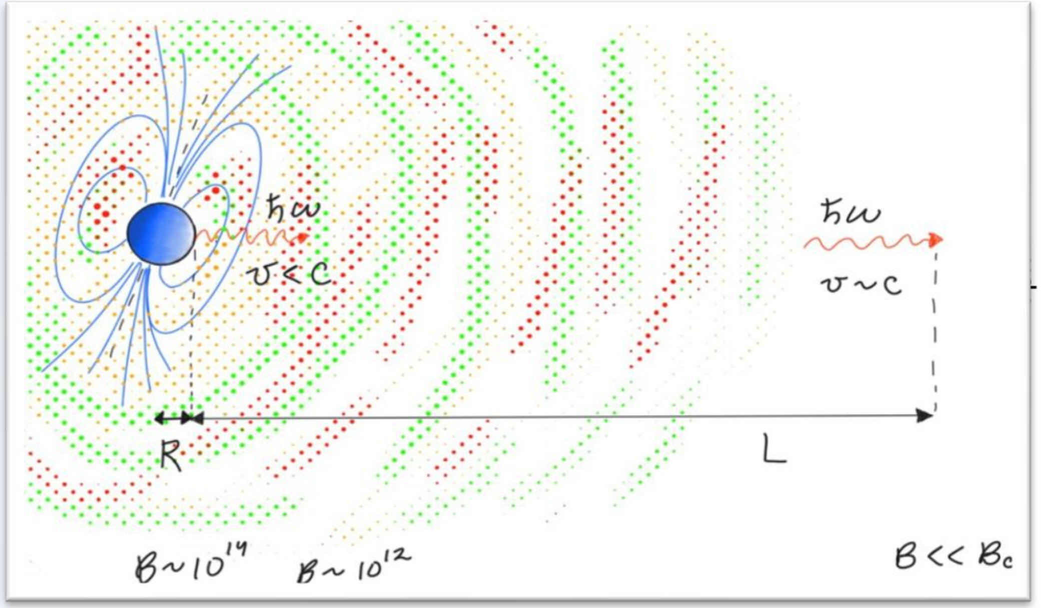
Parameters

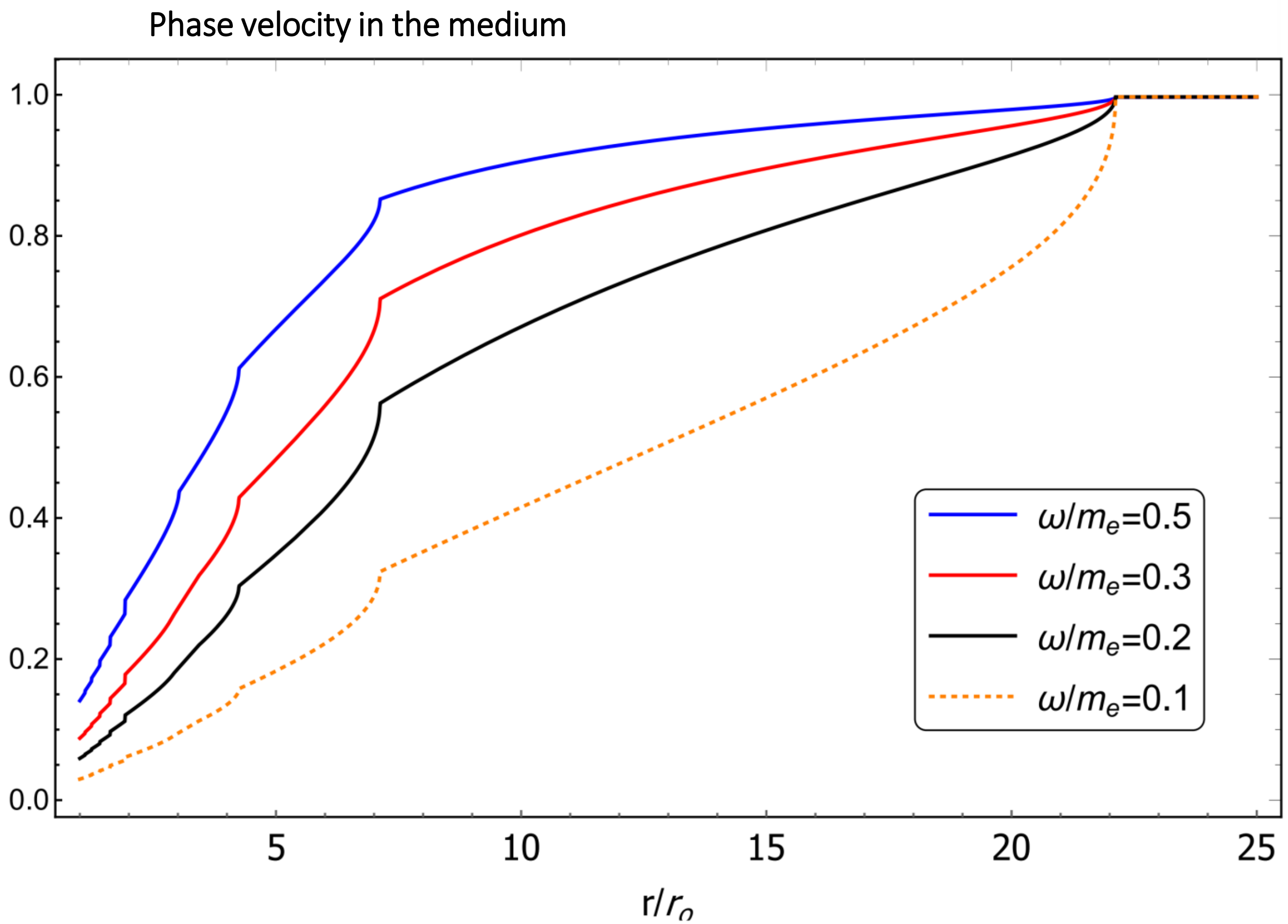
- $L = 25\, r_0$
- $\chi = \theta = \pi/6$
- $r_0 = 10\, km$
- $M_0 = 2\, M_s$
- $\mu_{min} = \mu_0 = 2\, m_e$
- $\mu_{max} = 50\, m_e$

$$\Delta\tau(\omega,L,M,B,\mu) = \int_R^{R+L} dr \left(\frac{1}{v_f(\omega,B,\mu)} - 1\right)\left(1 - \frac{2M}{r}\right)^{-1}$$

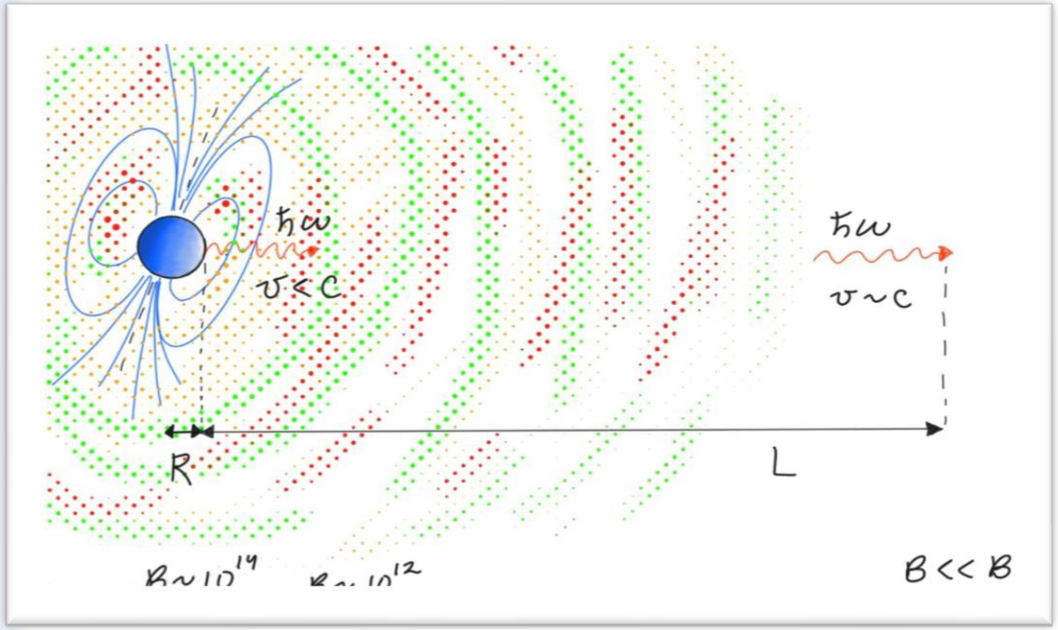
[m] *Bogovalov, 1999. Michel, 1991. Corotini 1990.* [n] *Pétri, 2011, 2016, 2018.*

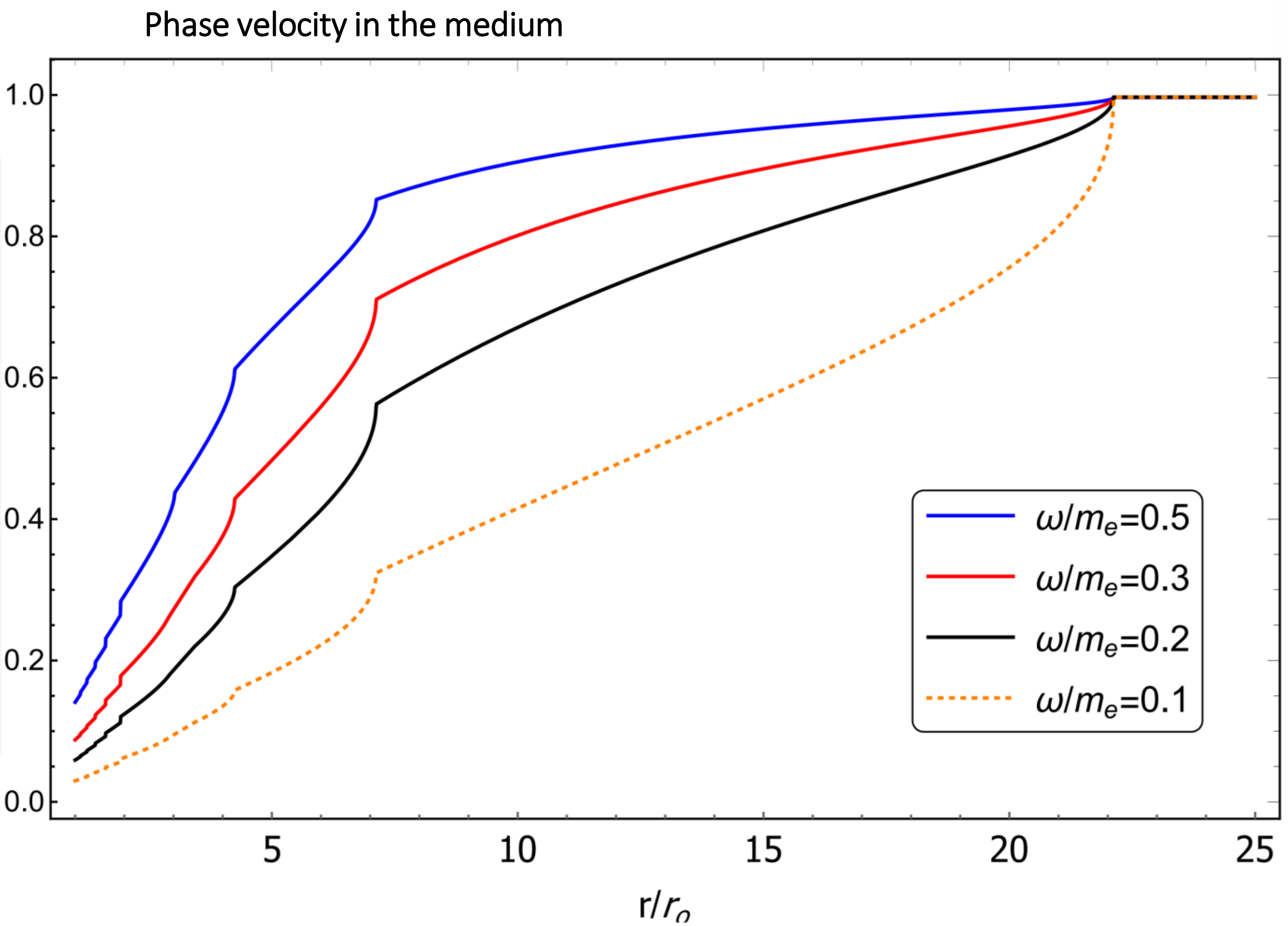
Photon Time Delay: Results





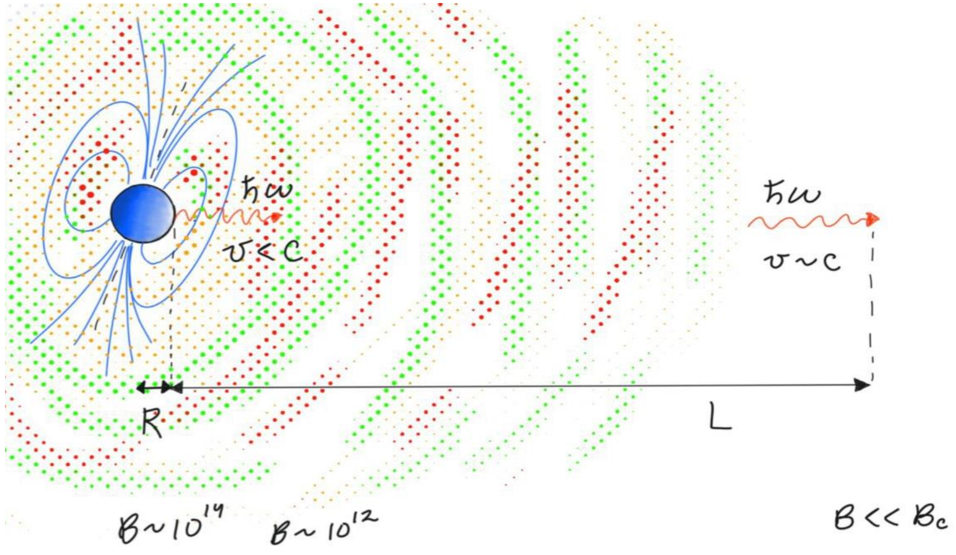
Photon Time Delay: Results



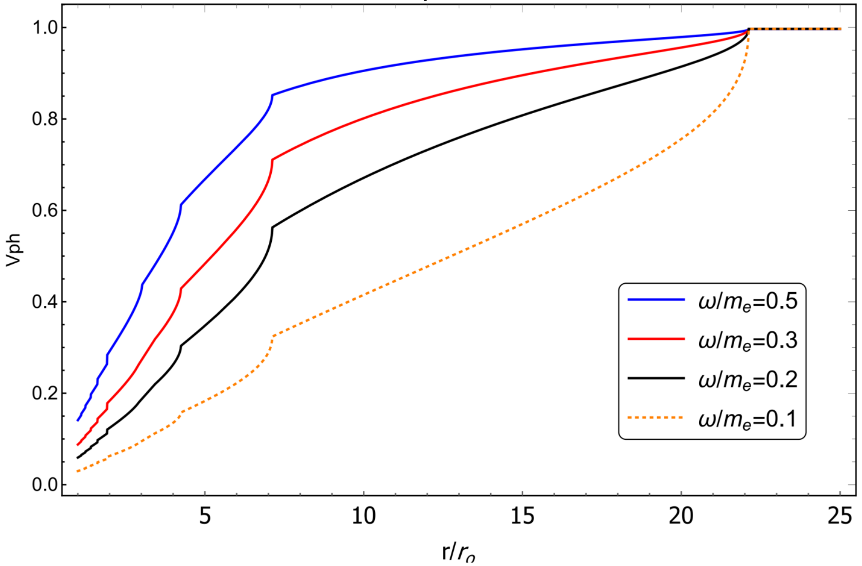


$$\Delta\tau(\omega, L, M, B, \mu) = \int_R^{R+L} dr \left(\frac{1}{v_f(\omega, B, \mu)} - 1\right)\left(1 - \frac{2M}{r}\right)^{-1}$$

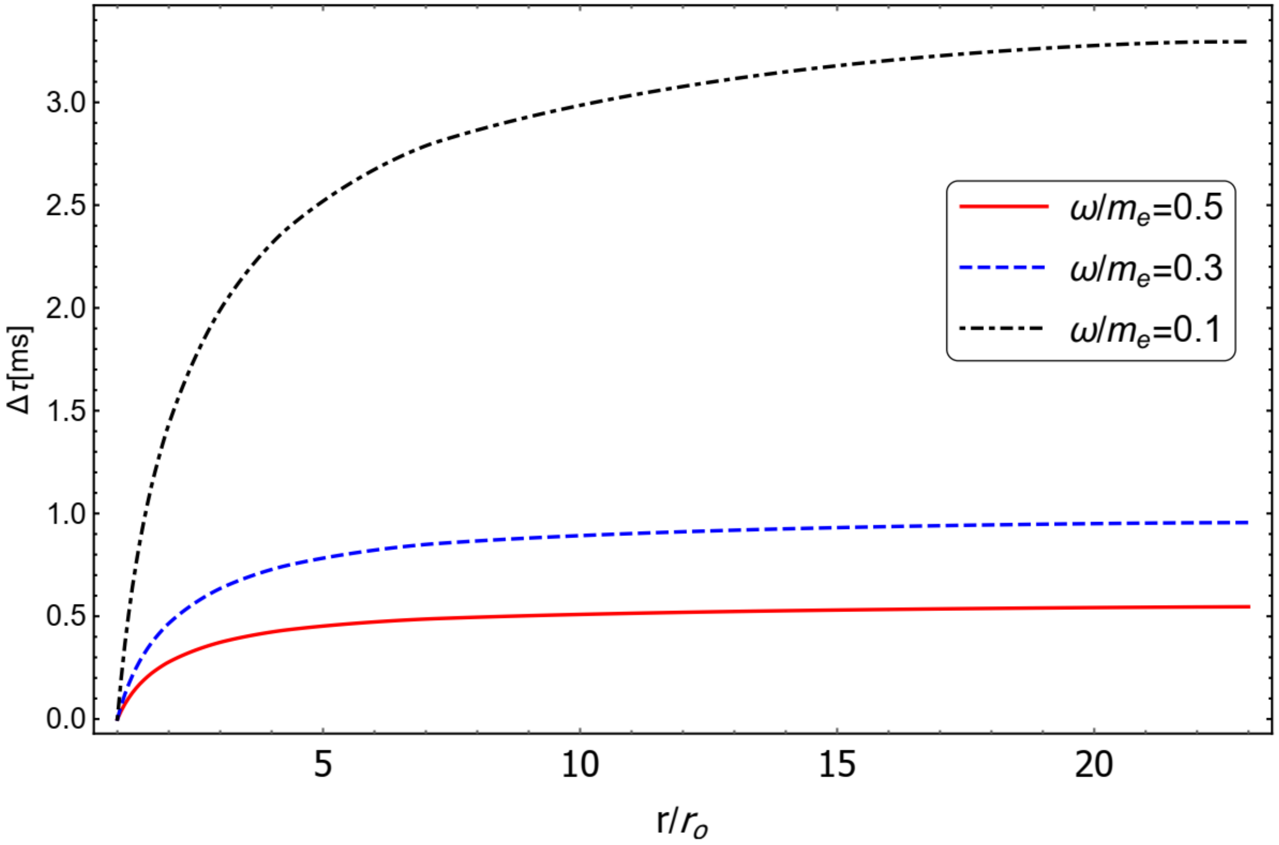
Photon Time Delay: Results



Phase velocity in the medium

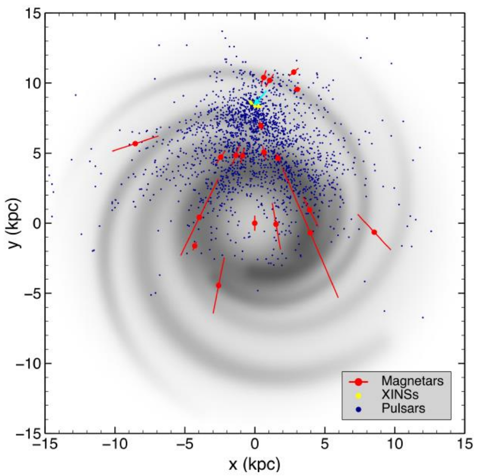


Time Delay in the medium



Photon Time Delay: Results

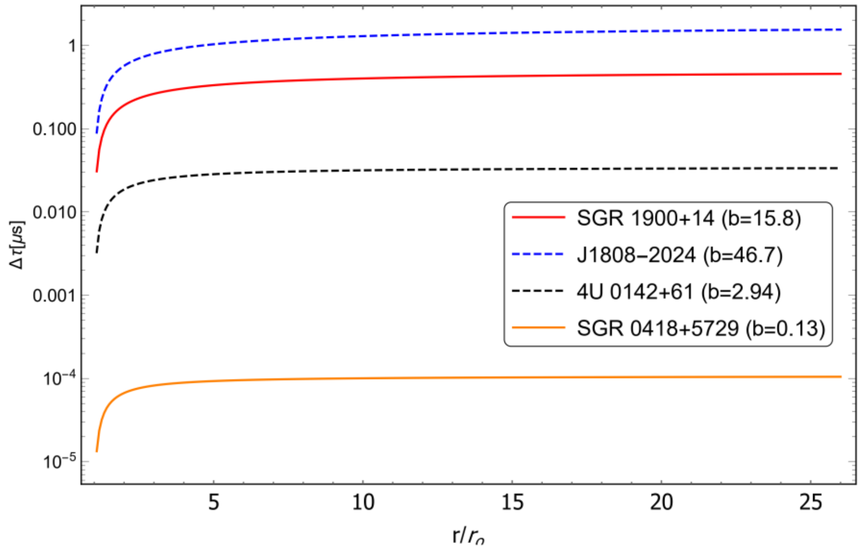
Olausen, 2014

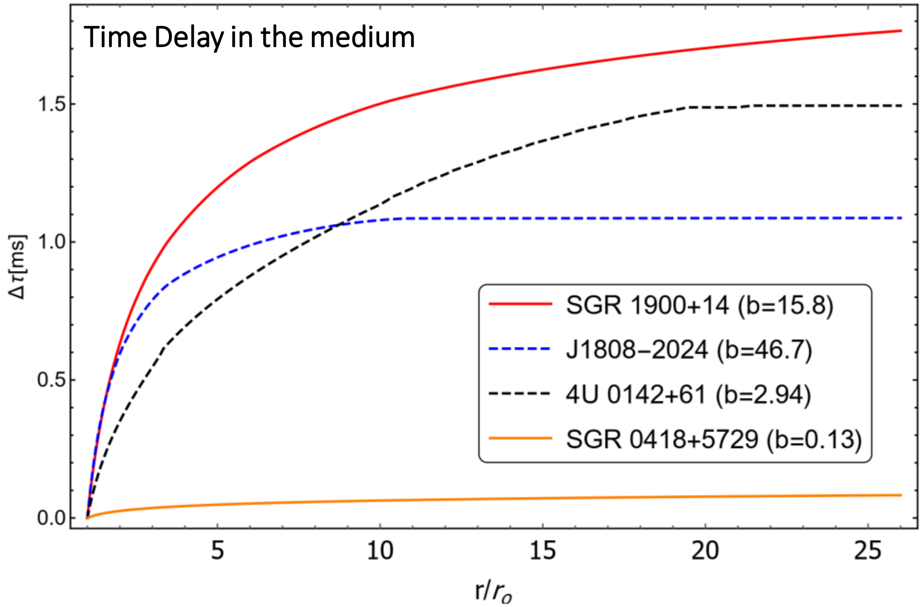


Púlsar	B_s (10^{14} G)	$\Delta\tau_V(ns)$
J010043.1-721134	3.9	180
4U 0142+61	1.3	33.5
SGR 0418+5729	0.061	0.104
SGR 0501+4516	1.9	61.6
J164710.2-455216	0.66	10.4
SGR 1900+14	7.0	401
1E 1547.0-5408	3.2	135
J1808-2024	20.6	1548

	$\Delta\tau_M(ms)$						
Pulsar	B_s (10^{14} G)	1 keV	5 keV	10 keV	50 keV	100 keV	500 keV
J010043.1-721134	3.9	123.7	24.0	11.6	1.92	0.99	9.2×10^{-3}
4U 0142+61	1.3	99.3	19.2	9.26	1.61	0.37	~ 0
SGR 0418+5729	0.061	24.8	3.00	0.86	~ 0	~ 0	~ 0
SGR 0501+4516	1.9	112.5	21.8	10.5	2.01	0.59	5.2×10^{-4}
J164710.2-455216	0.66	51.3	10.0	5.36	0.25	0.032	~ 0
SGR 1900+14	7.0	119.6	23.2	11.2	1.80	0.78	7.4×10^{-3}
1E 1547.0-5408	3.2	121.8	23.6	11.4	1.93	0.95	3.7×10^{-3}
J1808-2024	20.6	69.9	13.6	6.66	1.11	0.47	0.05

Time Delay in the vacuum





Photon Time Delay: Results

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	$\Delta\tau_M(ms)$						
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SGR 0418+5729	0.061	24.8	3.00	0.86	~ 0	~ 0	~ 0
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1E 1547.0-5408	3.2	121.8	23.6	11.4	1.93	0.95	3.7×10^{-3}
J1808-2024	20.6	69.9	13.6	6.66	1.11	0.47	0.05

Comparison with observational data

Cygnus X-2

$$\Delta\tau_{\text{Stollman}} \sim (1-4)\ ms$$

$$\omega \sim 1 - 10\ \text{keV}$$

$$\Delta\tau_{\text{teo}} \sim (1-10^2)\ ms$$

Stollman, 1987.

J1808-2024

$$\Delta\tau(2{,}3)_{\text{Denisov}} \sim 0.3\ \mu s$$

$$\Delta\tau(2{,}3)_{\text{teo}} = 1.40\ \mu s$$

Denisov, 2017

J0332+5434 *PSR B0531+21*

$$\Delta\tau \sim ms$$

Bentum, 2016.

Bhardwaj, 2016.

$\delta\tau \sim 10 - 100\ ns$ *Bailes, 2014.*

Photon Dispersion in Dirac materials

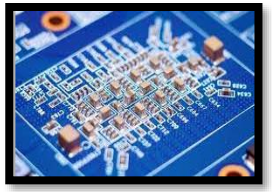
Brief introduction to the materials

Metals →
- High Conductivity
- Resistivity $\sim 10^{-8}\ \Omega \cdot m$

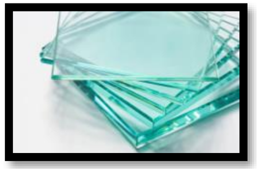


Semiconductors →
- Conductivity between metal and insulator
- Resistivity $\sim 10^{-6} - 10^{-4}\ \Omega \cdot m$

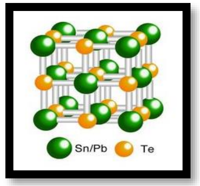




Insulators →
- Low Conductivity
- Resistivity $\sim 10^{8} - 10^{20} \Omega \cdot m$



Dirac Material →
- Relativistic fermions
- Lineal Relation energy-momentum

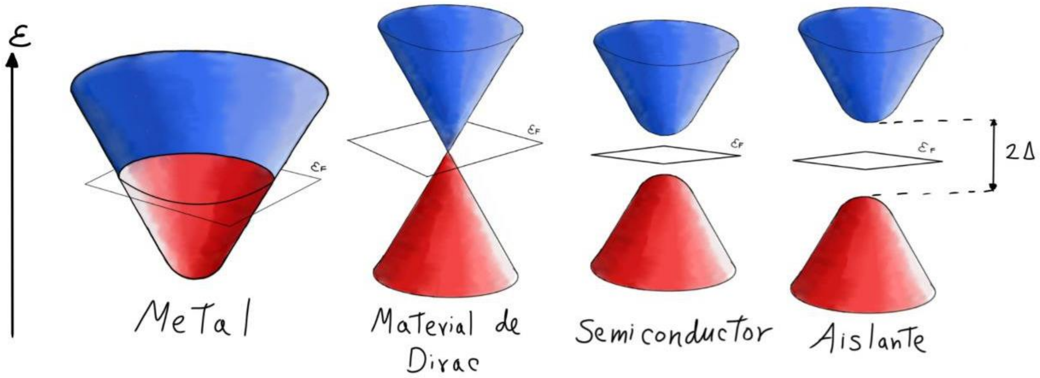


$Pb_{1-x}Sn_xTe$
$x = 0.2$

$Bi_{1-x}Sb_x$

Conduction Band

Valence Band

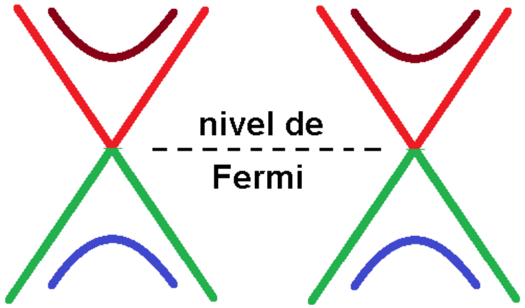


Lineal Energy $\varepsilon \approx v_F p$ $\quad \Delta = 0$

$v_F \approx 10^6 m/s$ $\quad \Delta \neq 0$

$\varepsilon, p \rightarrow$ Energy and momentum of the fermions
$v_F \rightarrow$ Fermi Velocity
$\Delta \rightarrow$ Band gap

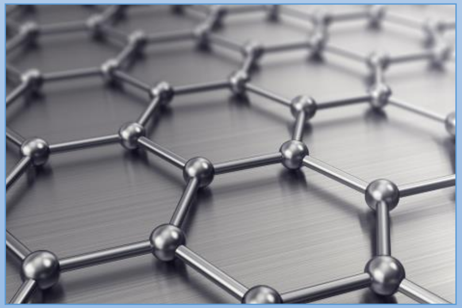
Dirac materials. Motivation.



$\Delta = 0$ High electrical conductivity, as in high-speed electronic devices.

$\Delta \neq 0$ Controlling the flow of electrons is required, especially in devices such as transistors, diodes and integrated circuits.

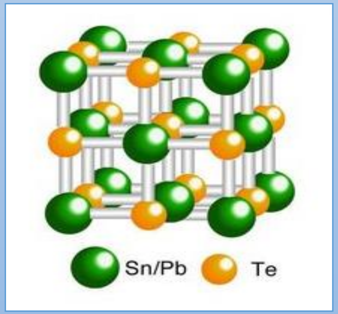
They are studied to test the effects of magnetized vacuum



Graphene [f]

$\mathbf{Ta_3As_5}$ [g]

$\mathbf{Bi_{1-x}Sb_x}$ [i]



$\mathbf{Pb_{1-x}Sn_xTe}$ [h]

[f] *Novoselov, 2004.* [g] *Kesser, 2021.* [h] *Dziawa, 2012.* [i] *Liu, 2008.*

Dirac materials. Model Considerations

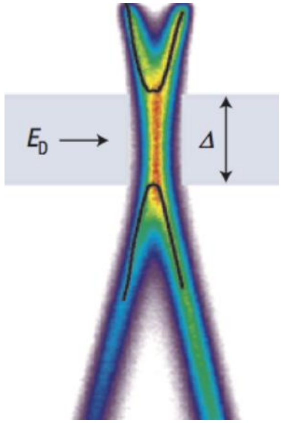
Simple Model

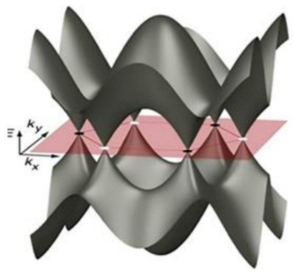
- All nodes are equal
- The energy gap is independent of the material characteristics and constant.

$$\Delta \neq 0 \qquad \Delta_1 = \cdots = \Delta_N$$

What defines a type of material?

$$v_F, \Delta \Rightarrow \alpha_D, B_c$$



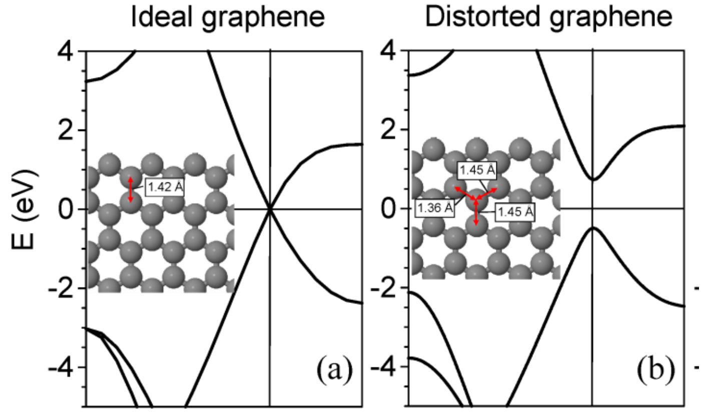


Graphene: 6 nodos

TaAs: 12 pares de nodos

PbSnTe: 14 (cub.) y 7 (hex.) nodos

BiSb: 19 nodos



$\Delta_{graf} = 100 - 250$ meV

DESY Hamburg, 2018

Villalba-Chávez, 2022

Sahu and Rout, 2017

$\Delta_{graf} = 100$ meV

Relativistic Electrons and Dirac Materials

$$\Delta \rightarrow m^* m_e v_F^2$$

$$c \rightarrow \mathrm{v_F}$$

$$\alpha = \frac{e^2}{\hbar c} \rightarrow \frac{e^2}{\hbar v_F}$$

$$v_F \approx 10^6 \mathrm{m/s}$$

$$E_c = \frac{m_e^2 c^3}{e\hbar} = 1{,}3 \times 10^{18}\ \mathrm{V/m} \rightarrow \sim 10^5\ \mathrm{V/m}$$

$$B_c = \frac{m_e^2 c^2}{e\hbar} = 4{,}4 \times 10^{13}\ \mathrm{G} \rightarrow \sim 10^4\ \mathrm{G} = 1\ \mathrm{T}$$

j,k

$E_c, B_c \rightarrow$ Critical Electric/Magnetic Field
$\alpha \rightarrow$ Fine Structure Constant
$\alpha_D \rightarrow$ Dirac Fine Structure Constant
$m^* \rightarrow$ Efective mass

	$\Delta(\mathrm{meV})$	α_D/α	B_c (T)	$b \leq 3\pi/\alpha_D$
EDC	10^9	1	4.4×10^9	1291
$Pb_{1-x}Sn_xTe$	31.5	580	5.6	2.22
$Bi_{1-x}Sb_x$	7.75	188	0.036	6.86
Ta_3As_5	21	357	0.95	3.61
Grafeno	100	301	15.4	4.28

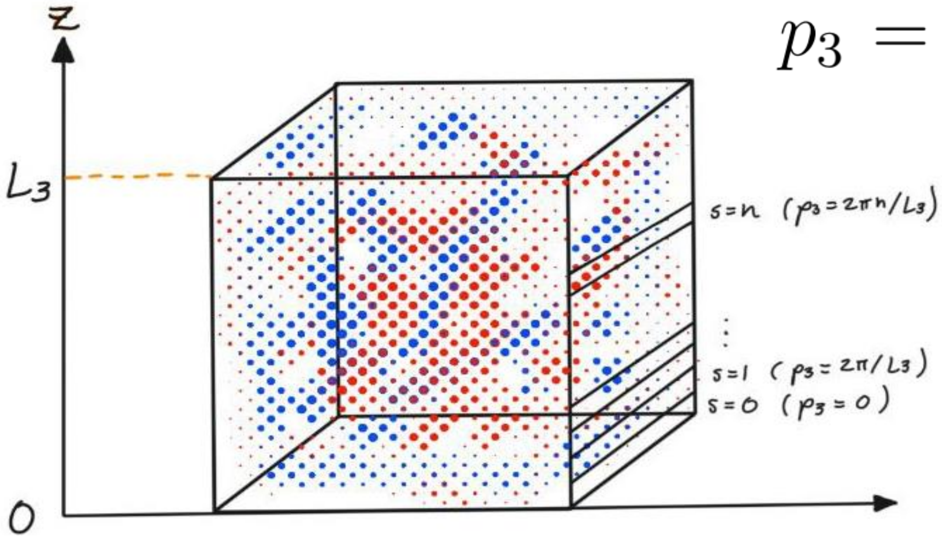
$$E_c(\Delta) = v_F B_c(\Delta) = \frac{\Delta^2}{e\hbar v_F}$$

$b = B/B_c(\Delta)$

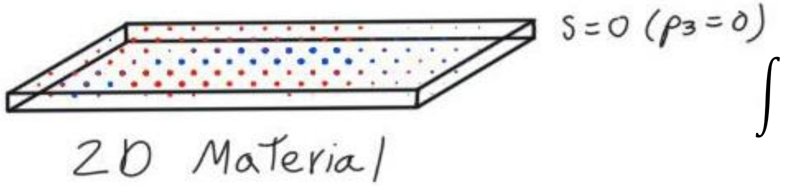
[j] Helayel, 2023. [k] *Kesser, 2021.*

Dimensional reduction 3D →2D.

$$p_3 = 2\pi s / L_3$$



Only modes with s = 0 are relevant, which effectively reduces the theory to two dimensions.



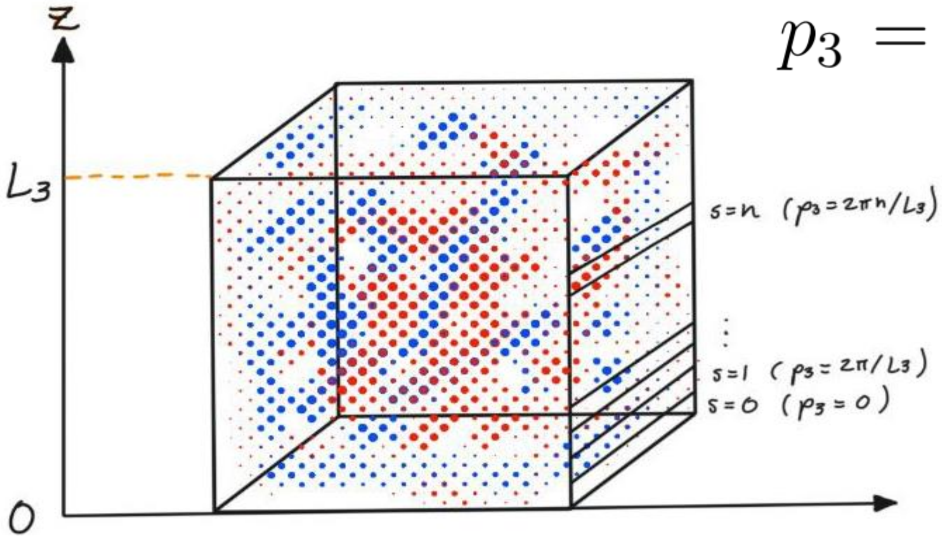
$$\int dp_3 \rightarrow \sum_{s=0}^{N} E_0$$

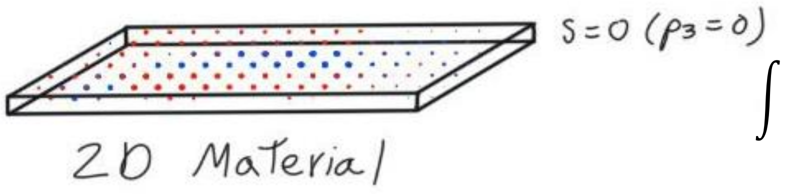
L_3 → Material Height
E_0 → Base State Energy

Dimensional reduction 3D →2D.

$$p_3 = 2\pi s / L_3$$

Only modes with s = 0 are relevant, which effectively reduces the theory to two dimensions.





$$\int dp_3 \to \sum_{s=0}^{N} E_0$$

L_3 → Material Height
E_0 → Base State Energy

Dispersion Law

- Small deviations $(p, r << t)$

$$\kappa^{(1)} = 0$$

$$\kappa^{(2)} = t = h - k_\perp^2 g$$

- Energy Spectrum $\varepsilon = \sqrt{\Delta^2 + 2en_l B v_F}$

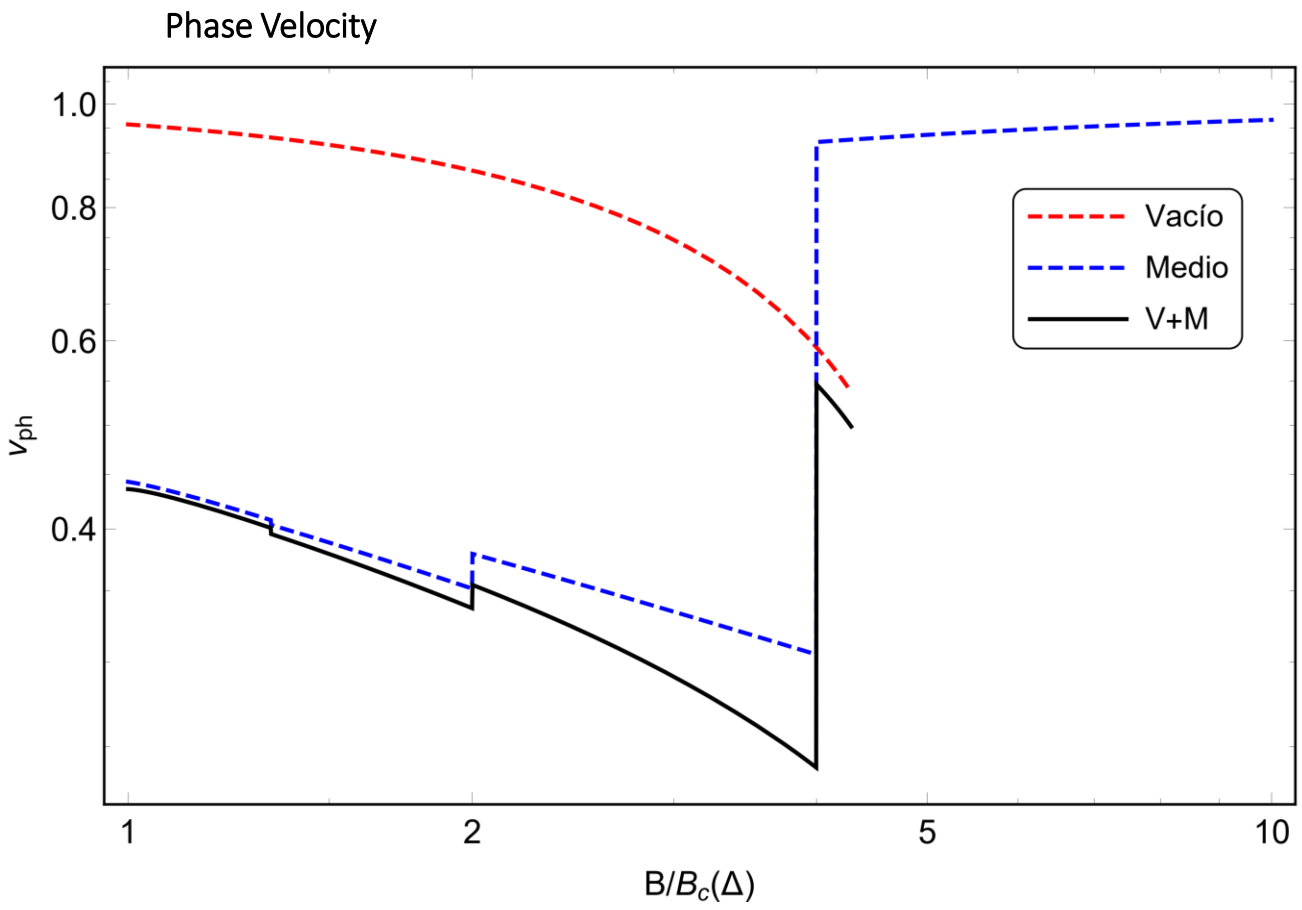
$$\kappa_M^{(2)} = -2\alpha_D eB \sum_{n_l=0}^{n_L} \frac{\Delta}{\varepsilon}\left[F_n^{(2)}(k_\perp)\left(1 - \frac{(2eB(2n+1)-\omega^2)(2eB-\omega^2)}{4\omega^2 p_0^2}\right)\right.$$
$$\left. -4N_n^{(1)}(k_\perp)\left(\frac{2eB-\omega^2}{4\omega^2 p_0^2}\right)\right].$$

- Relativistic electrons $\alpha \to \alpha_D$ $\quad c \to v_F$
- Degenerated gas limit $T \ll \mu$

$$n_L = I\left[\frac{\mu^2 - \Delta^2}{2eB}\right]$$

Results: Phase Velocity

$$k_\perp^2 - \omega^2 = \kappa_V^{(2)}(k_\perp, B) + \kappa_M^{(2)}(k_\perp, B, \omega, \mu, T) \qquad \kappa_V^{(2)} = \mathcal{L}_{GG} B^2 k_\perp^2$$



Parameters

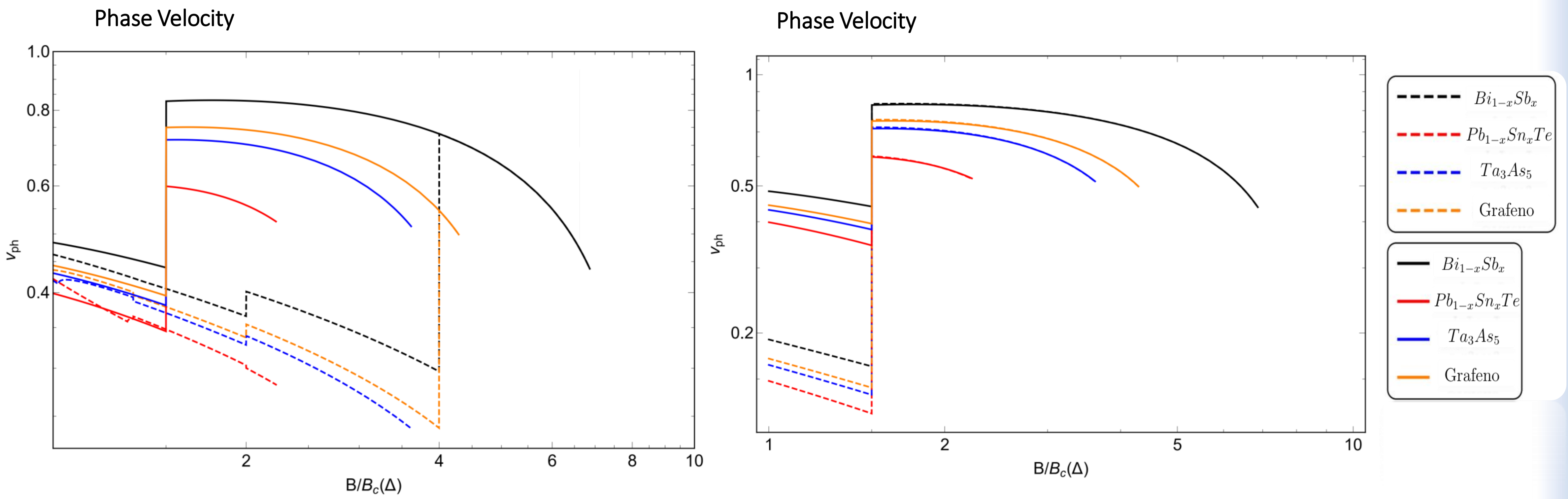
Graphene Type

$$\frac{\mu}{\Delta} = 3 \qquad \frac{\omega_1}{\Delta} = 0.1$$

$$B/B_c = 1$$

Results: Phase Velocity

$$k_\perp^2 - \omega^2 = \kappa_V^{(2)}(k_\perp, B) + \kappa_M^{(2)}(k_\perp, B, \omega, \mu, T)$$



Parameters

Graphene Type

$\frac{\omega_1}{\Delta} = 0.1$ $\frac{\mu_1}{\Delta} = 3$ (Continue)

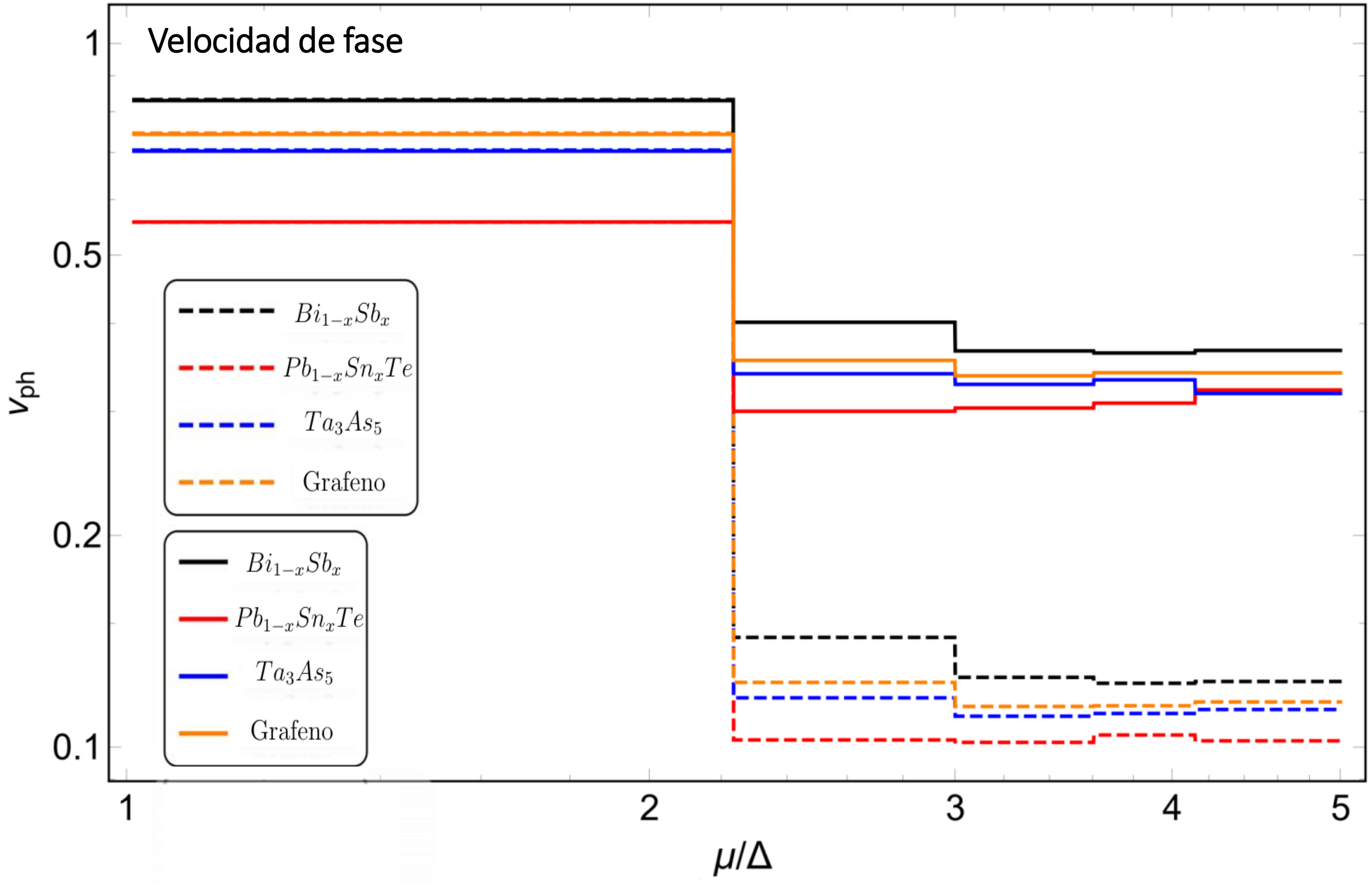
$\frac{\omega_2}{\Delta} = 0.01$ $\frac{\mu_2}{\Delta} = 2$ (Discontinue)

$\frac{\omega_1}{\Delta} = 0.1$ $\frac{\mu_1}{\Delta} = 2$ (Continue)

$\frac{\omega_2}{\Delta} = 0.01$ $\frac{\mu_2}{\Delta} = 2$ (Discontinue)

Resultados: velocidad de fase

$$k_{\perp}^2 - \omega^2 = \kappa_V^{(2)}(k_{\perp}, B) + \kappa_M^{(2)}(k_{\perp}, B, \omega, \mu, T) \qquad \kappa_V^{(2)} = \mathcal{L}_{GG} B^2 k_{\perp}^2$$



Parámetros

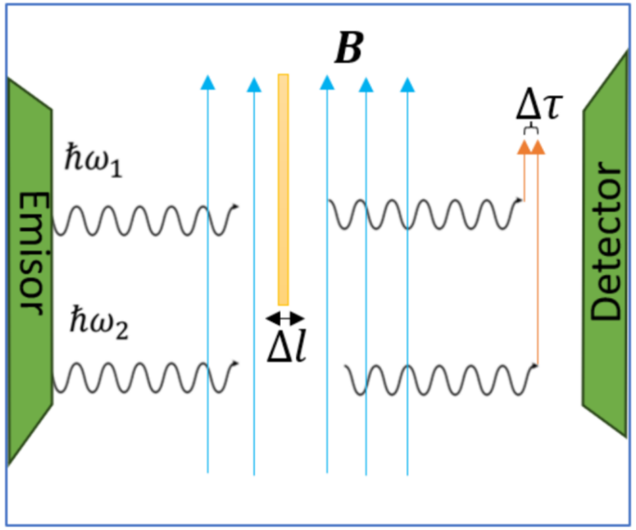
Graphene Type

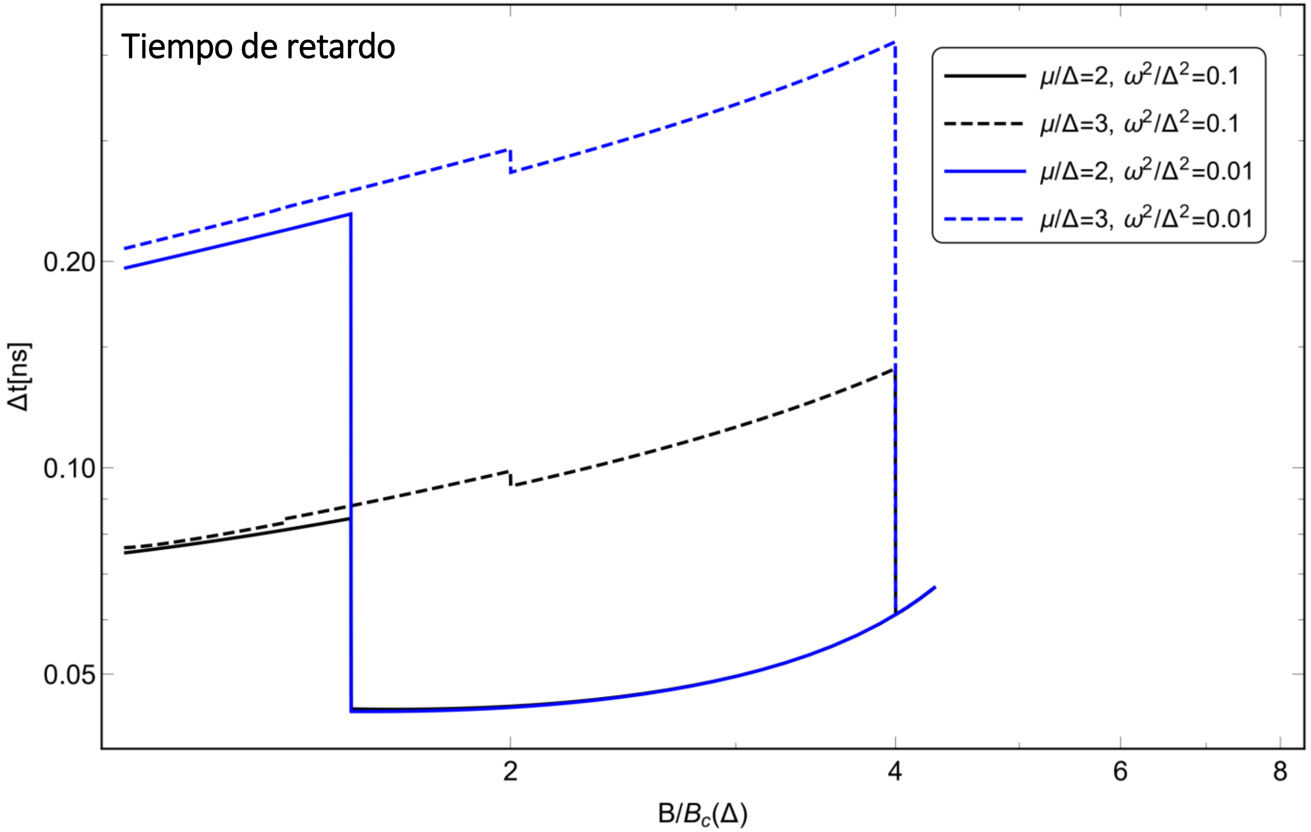
$\frac{\omega_1}{\Delta} = 0.1$ (Continue)

$\frac{\omega_2}{\Delta} = 0.01$ (Discontinue)

$B/B_c = 2$

Resultados: Tiempo de Retardo





Parameters

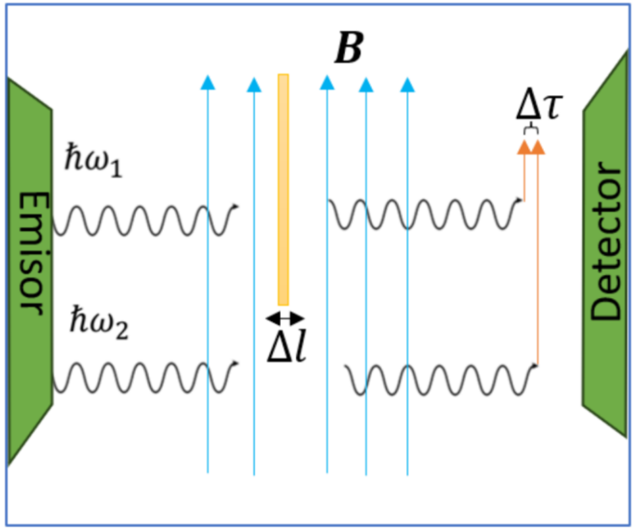
- $\Delta l = 1\, mm$
- $\omega_1/m_e = 0.1/0.01$
- $B/B_C(\Delta) = 1/2$
- $\mu/m_e = 2/3$

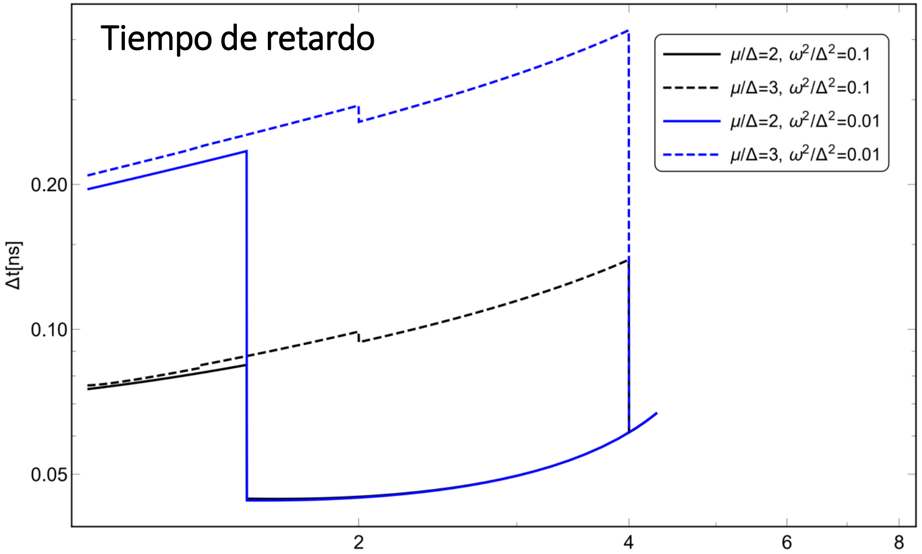
$$\Delta\tau(\omega,\mu,B) = \frac{\Delta l}{v_f(\omega,\mu,B)} - \Delta l$$

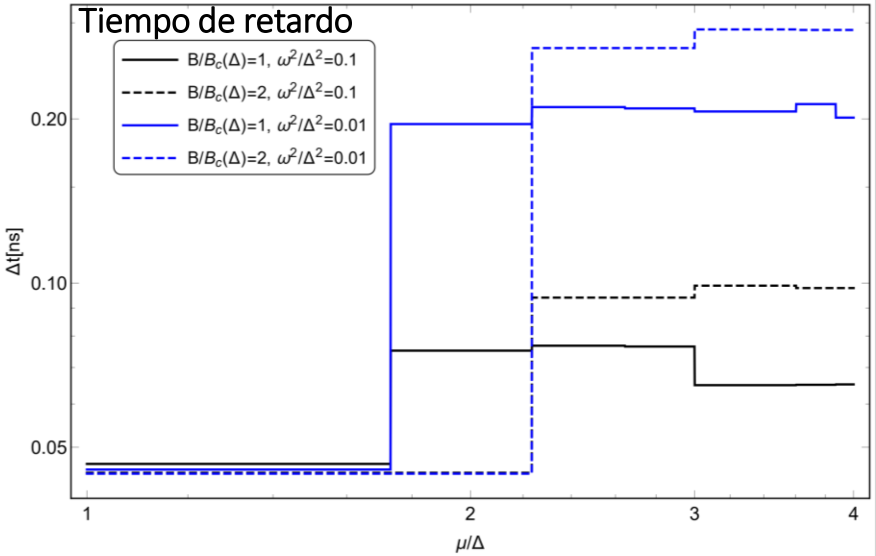
Graphene Type

$\Delta\tau < ns$

Resultados: Tiempo de Retardo







$$\Delta\tau(\omega,\mu,B) = \Delta l\left(\frac{1}{v_f(\omega,\mu,B)} - 1\right)$$

Graphene Type

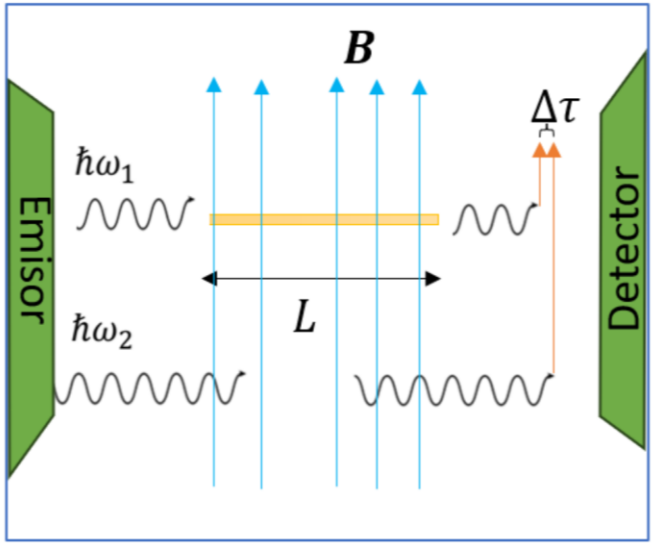
$\Delta\tau < ns$

$\delta\tau \sim 10 - 100\, ns$

Parameters

- $\Delta l = 1\, mm$
- $\omega_1/m_e = 0.1/0.01$
- $B/B_C(\Delta) = 1/2$
- $\mu/m_e = 2/3$

Resultados: Tiempo de Retardo



Parameters

- $L = 1\,m$
- $\omega_1/m_e = 0.1/0.01$
- $B/B_C(\Delta) = 1/2$
- $\mu/m_e = 2/3$

$$\Delta\tau(\omega,\mu,B) = L\left(\frac{1}{v_f(\omega,\mu,B)} - 1\right)$$

$$\Delta\tau = 10^2\,ns$$

$$\delta\tau \sim 10 - 100\,ns$$

Possible experimental setups

- Use an array of N parallel two-dimensional plates in series
- Use mirrors in the emitter and detector to reflect the wave

Conclusions

1. The perpendicular propagation of photons with respect to an intense magnetic field was studied, using QED and nonlinear QED. An analytical dispersion law was obtained in the degenerate gas limit.
2. Two physical scenarios of interest were studied: The magnetosphere of neutron stars and Dirac materials.
3. The phase velocity for photons emitted from the pulsar surface was obtained using the proposed magnetized medium model.
4. The time delay for some magnetars of interest was calculated for photons in the X-ray and gamma ray spectrum, for both the vacuum and medium contributions.
5. They were compared with the experimental data and are in the expected order. Additionally, higher energy photons experience a shorter time delay.
6. Photon propagation was studied in novel two-dimensional graphene-type Dirac materials. The dispersion law containing the contributions of the vacuum and the medium was obtained.
7. The phase velocity and delay time were calculated in the case of photon bombardment against a 2D sheet of the material.

Recommendations

For neutron stars:

1. Estimate the distance from the radiation source by measuring the different delay times. Build a method to measure the distance to pulsars.
2. Build a more complete model to compare theoretical results with observational data. Consider the additional delay that photons experience as they move away from the neutron star due to the nebula arm and the interaction with the interstellar remnant on their journey to Earth.
3. Consider the mass-radius values that have been obtained from models that take the magnetic field into the equations of state for some pulsars to improve the model when calculating the delay time.

For Dirac materials:

1. Include the dependence of the forbidden gap on temperature, chemical potential, Landau levels and other material parameters.
2. Additionally, consider the node structure of each material.

Thanks for your attention

¨ Theoretical and experimental physicists are studying nothingness, the vacuum. But that nothing contains everything¨, Heinz R. Pagels

Preguntas del Oponente

Pregunta 1:

¿La velocidad de fase de los fotones que se propagan en el medio considerado en un campo magnético, mayor al crítico, siempre es menor que la velocidad de la luz ? ¿Qué sucede con la velocidad de grupo en el vacío y en el medio?

$$v_f = \frac{\omega}{k_\perp} \qquad v_f < 1$$

$$v_g = \frac{\partial \omega}{\partial k_\perp} = \frac{1}{n + \omega(\partial n/\partial \omega)}$$

Vacío $\frac{\partial n(B)}{\partial \omega} = 0 \qquad v_f^V = v_g^V$

Medio $\frac{\partial n(k_\perp, \omega, B, T)}{\partial \omega} \neq 0 \qquad v_f^V \neq v_g^V$

$c = 299\,792{,}458\, m/s \qquad n = v_f^{-1}$

Shabad, 2021.

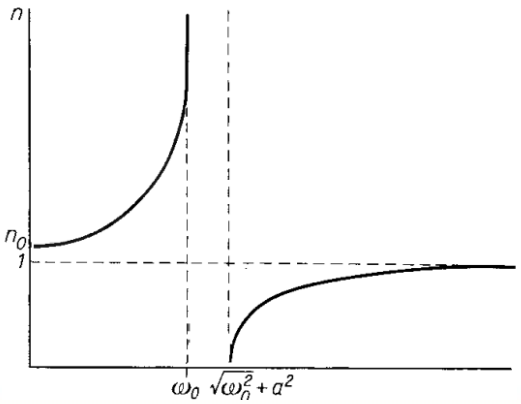
Brillouin (1946) "que en un medio, si también se produce absorción, la velocidad del grupo deja de tener un significado físico claro."

$$v_g = \left(\frac{\partial(Re\{k_\perp\})}{\partial \omega}\right)^{-1}$$

Dispersión normal $\partial n/\partial \omega > 0 \qquad v_g < v_f < 1$

Dispersión anómala $\partial n/\partial \omega < 0$

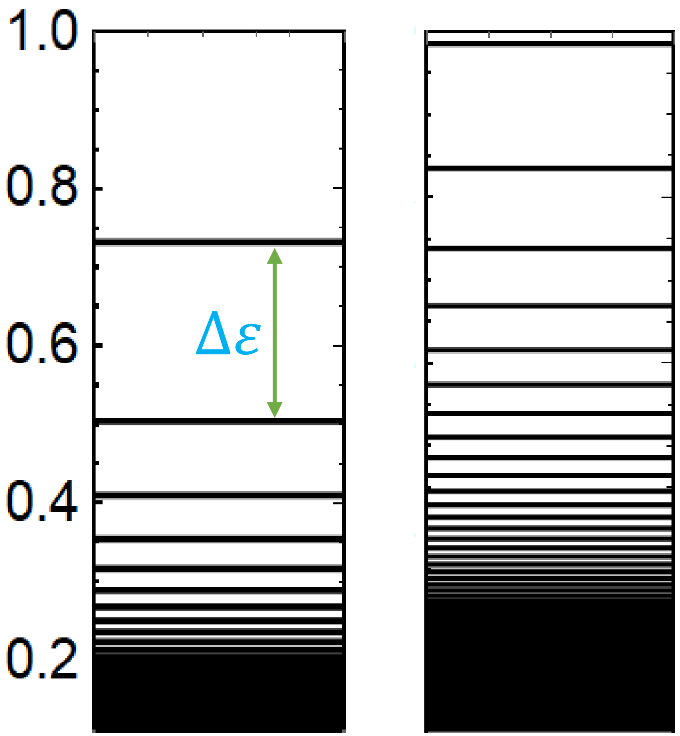
$v_g > 1, < 0, \to \infty$



Vg puede no ser una cantidad bien definida o puede no ser una cantidad significativa.

Pregunta 2:

¿Por qué la velocidad de fase en el medio magnetizado posee una tasa de cambio mayor en el rango menos energético para los fotones? ¿Por qué se dice que al aumentar el campo magnético por encima del valor crítico la velocidad de los fotones tiende a la velocidad de la luz, si al aumentar este es donde se observa la dispersión más fuerte (Figura 1.5)?



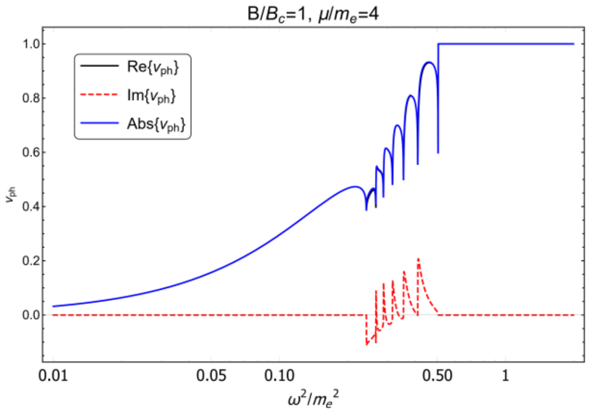
$B/B_c = 1$ $B/B_c = 5$

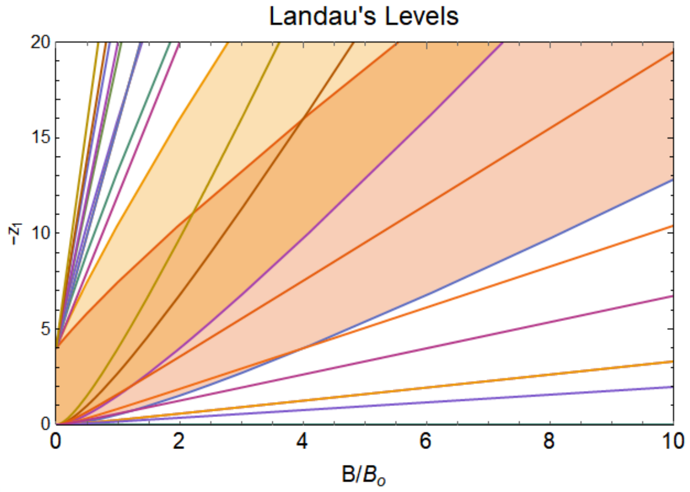
$$\varepsilon_{0,n_l} = \sqrt{m_e^2 + 2eBn_l}$$

$$\Delta\varepsilon = |\varepsilon_{0,n_f} \pm \varepsilon_{0,n_i}|$$

Los fotones de menor energía tienen una probabilidad mayor de ser absorbidos o emitidos por el medio

$$\omega \sim \Delta\varepsilon$$





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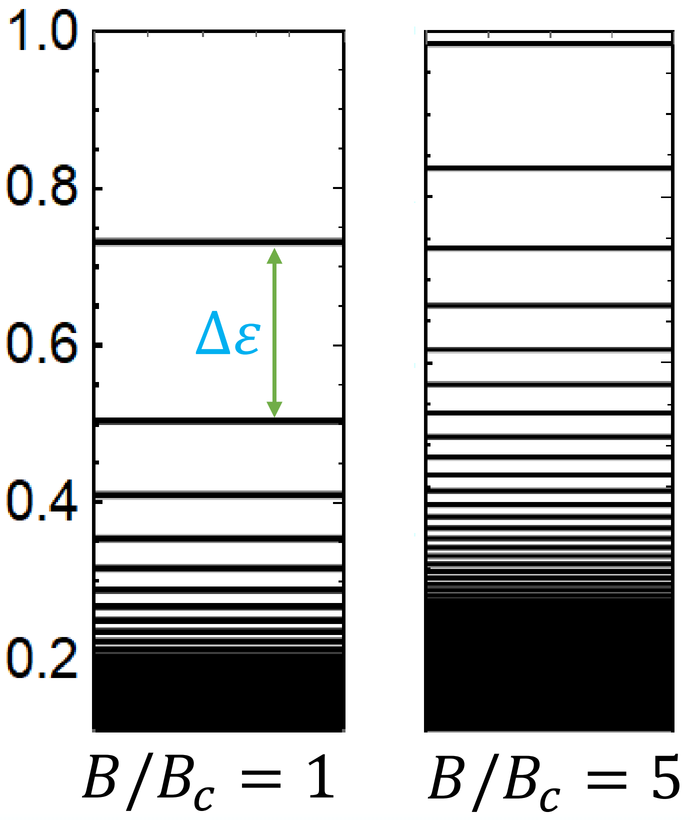


$$\varepsilon_{0,n_l} = \sqrt{m_e^2 + 2eBn_l}$$

$$\Delta\varepsilon = |\varepsilon_{0,n_f} \pm \varepsilon_{0,n_i}|$$

Los fotones de menor energía tienen una probabilidad mayor de ser absorbidos o emitidos por el medio

$$\omega \sim \Delta\varepsilon$$

$$\kappa \sim \frac{1}{p_0} \qquad p_0 = \frac{A}{2\omega}$$

$$A = ((\omega^2 - \omega^{2,\prime})(\omega^2 - \omega^{2,\prime\prime}))^{1/2}$$

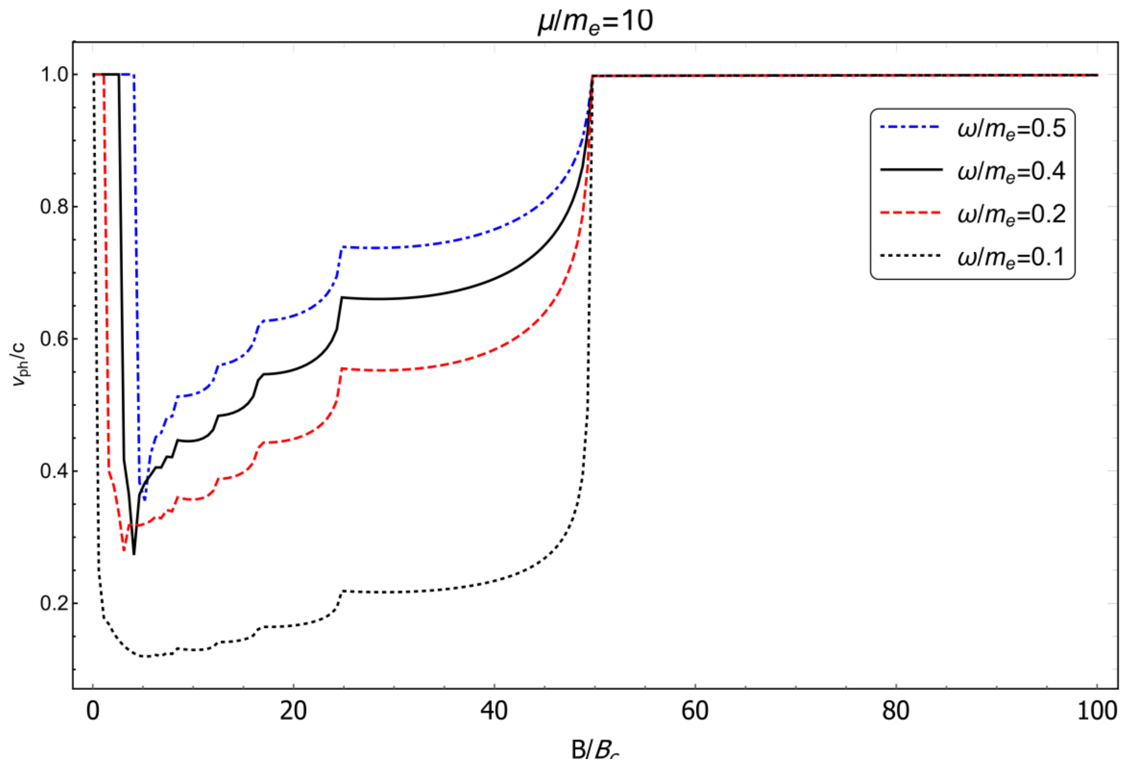
$$\omega' = \Delta\varepsilon_+ \qquad \omega'' = \Delta\varepsilon_-$$

$$\lim_{\omega\to\omega',\omega''} p_0 \sim 0$$

$$\lim_{\omega\to\omega',\omega''} \kappa \gg 1 \Rightarrow v_f \ll 1$$

Pregunta 2:

¿Por qué la velocidad de fase en el medio magnetizado posee una tasa de cambio mayor en el rango menos energético para los fotones? ¿Por qué se dice que al aumentar el campo magnético por encima del valor crítico la velocidad de los fotones tiende a la velocidad de la luz, si al aumentar este es donde se observa la dispersión más fuerte (Figura 1.5)?



$$n_L = 0 \quad 0 < \left[\frac{\mu^2 - m_e^2}{2eB}\right] = \left[\frac{\mu'^2 - 1}{2b}\right] < 1$$

$$b = B/B_c$$

$$1 < B/B_c < \frac{\mu^2 - 1}{2} = 49{,}5$$

Se observa la dispersión más fuerte

$$\frac{\mu^2 - 1}{2} < B/B_c$$

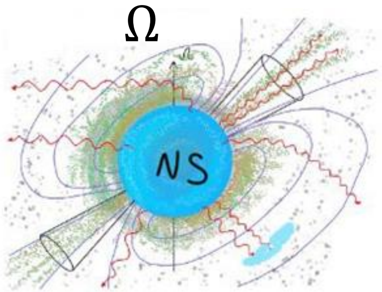
La velocidad de los fotones tiende a un valor cercano a 1

Pregunta 3:

En cuanto al modelo considerado para la magnetósfera de los púlsares, ¿por qué no usar la métrica de Kerr en vez de la de Schwarzchild, si los efectos de rotación son esenciales para describir la dinámica de los púlsares? ¿Por qué dice además que el campo magnético es toroidal en todo el espacio?

Métrica de Schwarzchild

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d^2\theta + \sin^2\theta d^2\phi)$$



$T \sim 10^{-2} - 10^{1}$ s $\qquad \Delta T = 1\, ms$

$c \sim 10^6\, m/s \qquad \Delta L = 30\, r_0$

Métrica de Kerr

- Forma Elipsoidal, Rotación

$$\Sigma = r^2 + \frac{a^2}{c^2}\cos^2\theta,$$

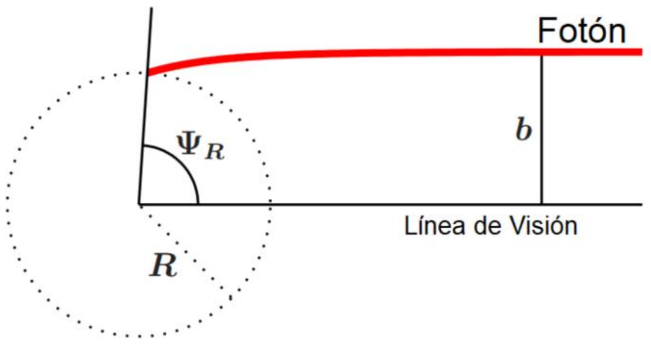
$$ds^2 = -\left(1 - \frac{2GMr}{c^2\Sigma}\right) c^2 dt^2 - \frac{4aGMr\sin^2\theta}{c^3\Sigma} cdtd\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + \frac{a^2}{c^2} + \frac{2a^2 Mr\sin^2\theta}{c^4\Sigma}\right)\sin^2\theta d\phi^2 \qquad \Delta = r^2 - \frac{2GMr}{c^2} + \frac{a^2}{c^2}$$

Campo magnético toroidal

- Forma Dipolar, poloidal, toroidal

Pregunta 4:

¿Cómo se tiene en cuenta en el cálculo del tiempo de retardo la curvatura del espacio-tiempo, si se consideran trayectorias rectilíneas?, ¿por qué considerar el parámetro de impacto cero en el cálculo?



$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2(d^2\theta + \sin^2\theta d^2\phi)$$

$$\frac{\mathrm{d}t}{\mathrm{d}\lambda} = (1 - r_s/r)^{-1} \qquad \frac{\mathrm{d}r}{\mathrm{d}\lambda} = \left[1 - b^2\left(1 - r_s/r\right)/r^2\right]^{1/2}$$

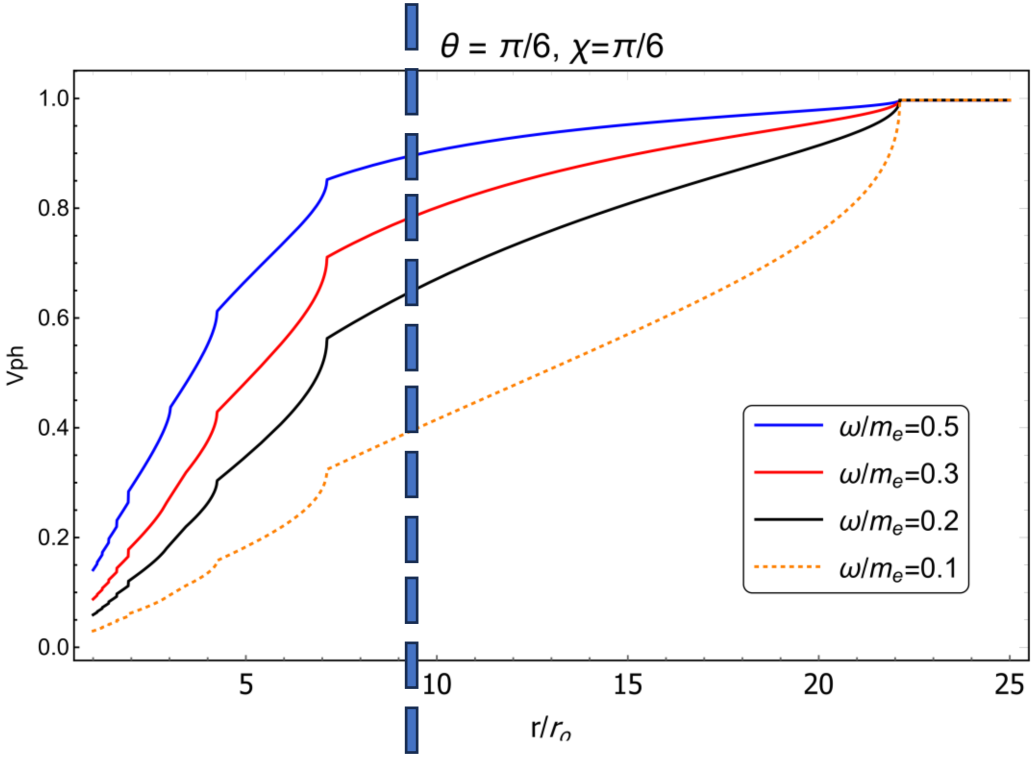
$$t_v = \int_{t(R)}^{t(\infty)} dt = \int_R^\infty \frac{dt}{d\lambda}\frac{d\lambda}{dr}\frac{dr}{v} = \int_R^\infty \frac{dr}{v}\left(1 - \frac{2M}{r}\right)^{-1}\left[1 - \frac{b^2}{r^2}\left(1 - \frac{2M}{r}\right)\right]^{-1/2}$$

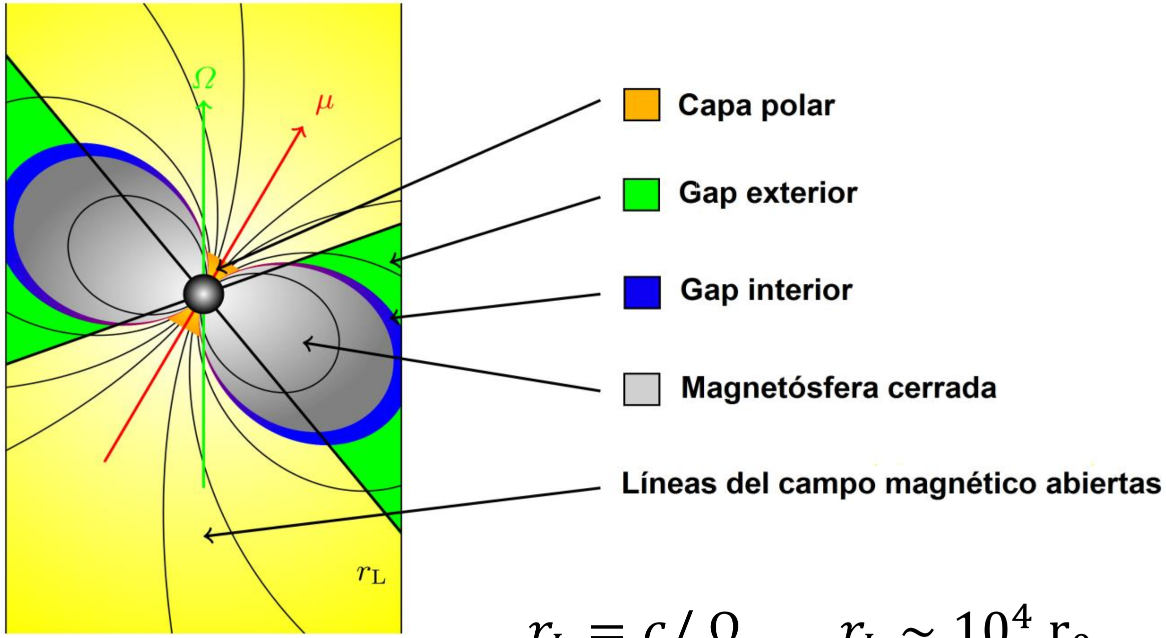
$$\Delta\tau(\omega, L, M, \mu, B) = \int_{r_0}^{r_0+L} dr\left(\frac{1}{v_f(\omega,\mu,B)} - 1\right)\left(1 - \frac{2M}{r}\right)^{-1}$$

Se utilizan trayectorias radiales para el fotón

Pregunta 5:

Discutir en la fig. 2.4 la región de la magnetósfera donde ocurre la mayor dispersión





$r_L = c/\,\Omega \qquad r_L \sim 10^4\ \mathrm{r}_0$

$$\frac{B(r=10\,r_0)}{B(r=r_0)} = 0.10 \qquad \frac{\mu(r=10\,r_0)}{\mu(r=r_0)} = 0.13$$

Región de a magnetósfera donde ocurre la mayor dispersión $r/r_0 < 10$

Pregunta 6:

Definir la estructura de banda a partir de la relación de dispersión en el modelo considerado, ¿qué sucede si el potencial químico se encuentra entre la banda de conducción y la banda de valencia?

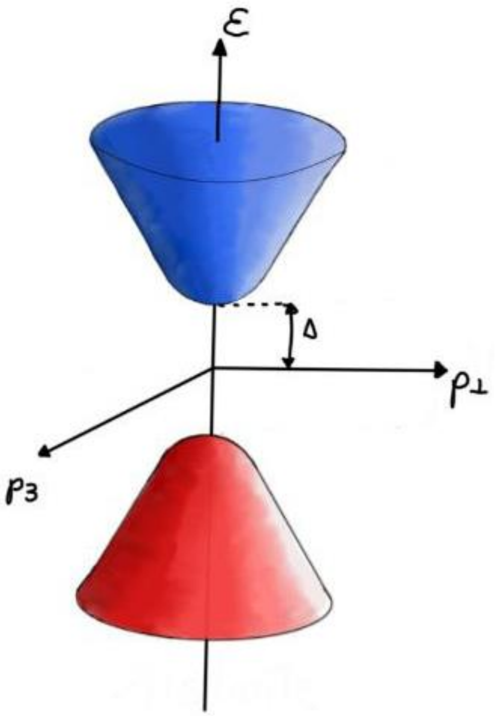
$$\varepsilon = \pm\sqrt{p_3^2 v_F^2 + \Delta^2 + p_\perp^2 v_F^2} \qquad p_\perp^2 = 2eBn_l$$

Reducción dimensional

$$\varepsilon = \pm\sqrt{\Delta^2 + p_\perp^2 v_F^2}$$

con $\Delta = 0$

$\varepsilon = \pm v_f p_\perp$ Relación Lineal



Límite de gas degenerado

$$p_F = \sqrt{\mu^2 - \Delta^2 - 2eBn_l} > 0$$

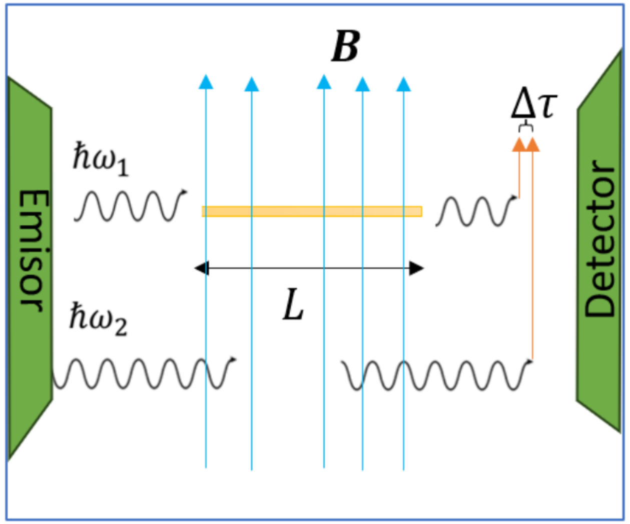
$$n_L = I\left[\frac{\mu^2 - \Delta^2}{2eB}\right] \geq 0 \qquad \mu \geq \Delta$$

Hay electrons libres de conducción

$$\kappa_M^{(2)} = -2\alpha_D eB \sum_{n_l=0}^{\infty} \frac{\Delta}{\varepsilon}\left[F_n^{(2)}(k_\perp)\left(1 - \frac{(2eB(2n+1) - \omega^2)(2eB - \omega^2)}{4\omega^2 p_0^2}\right) - 4N_n^{(1)}(k_\perp)\left(\frac{2eB - \omega^2}{4\omega^2 p_0^2}\right)\right](n_e + n_p)$$

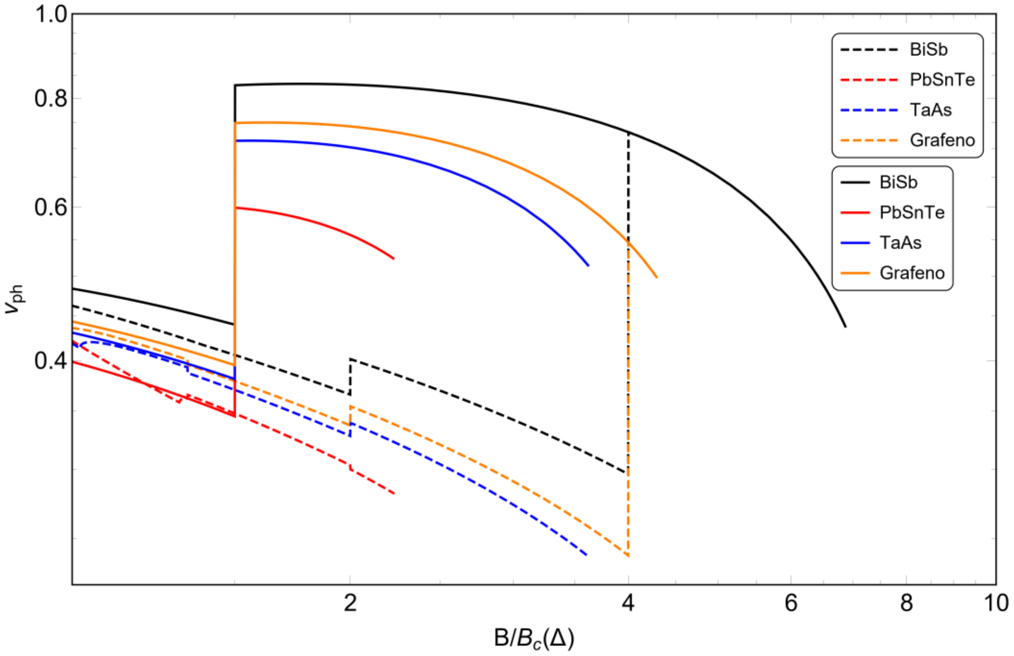
Pregunta 7:

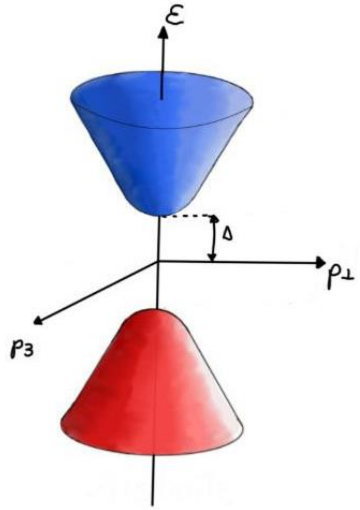
En el montaje experimental considerado, ¿cómo se tiene en cuenta que la lámina en un material de Dirac?



$$v_f = \omega/k_\perp \qquad k_\perp^2 - \omega^2 = \kappa_V(k_\perp, B) + \kappa_M(k_\perp, B, \mu)$$

$$\varepsilon = \pm\sqrt{\Delta^2 + 2eBn_l}$$





Gracias por su atención

¨Los físicos teóricos y experimentales están estudiando la nada, el vacío. Pero esa nada lo contiene todo¨, Heinz R. Pagels

Índice de Contenidos

Ley de Dispersión. Materiales de Dirac 2D

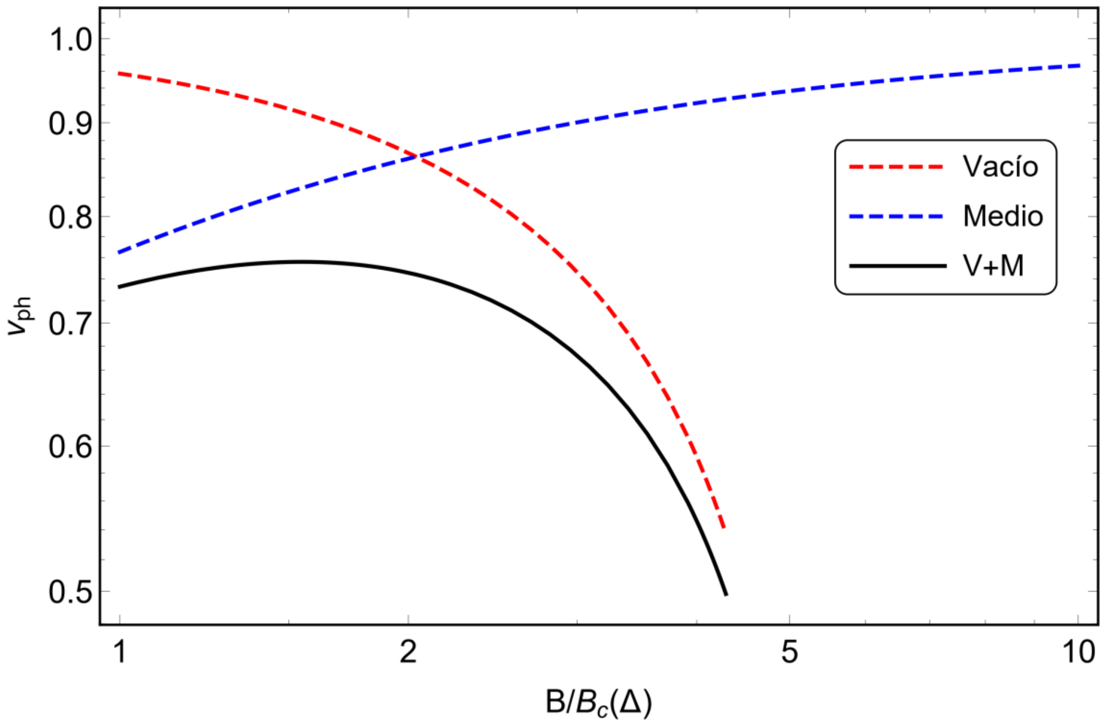
Campo Fuerte, n=0

$$\varepsilon = \Delta \qquad F^{(2)}_{n_l n_l'} \to e^{-k_\perp^2/2eB} \approx 1 - k_\perp^2/2eB \qquad N_{n_l n_l'} \to 0$$

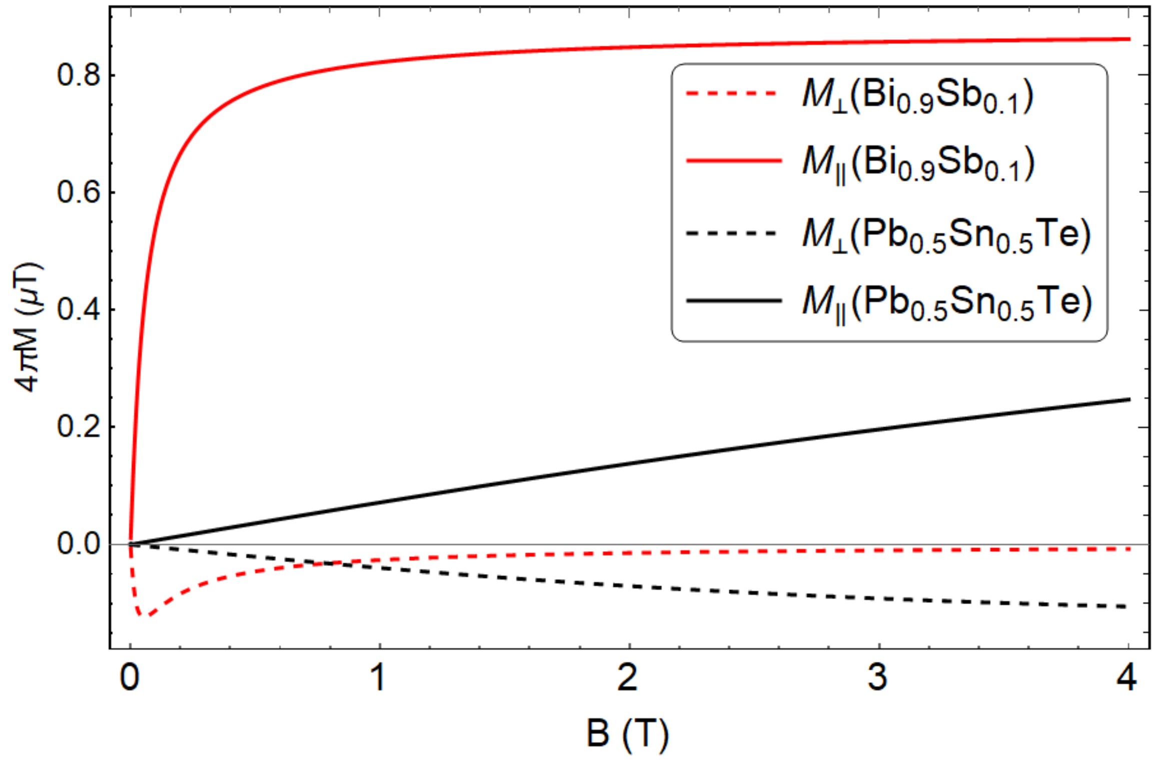
$$\kappa^{(2)} = -\frac{e^3 B}{4\pi^2}\left(1 - \frac{k_\perp^2}{2eB}\right)\frac{1}{\varepsilon}\left(1 - \frac{(2eB-\omega^2)^2}{4\omega^2 p_0^2}\right)(n_e + n_p)$$

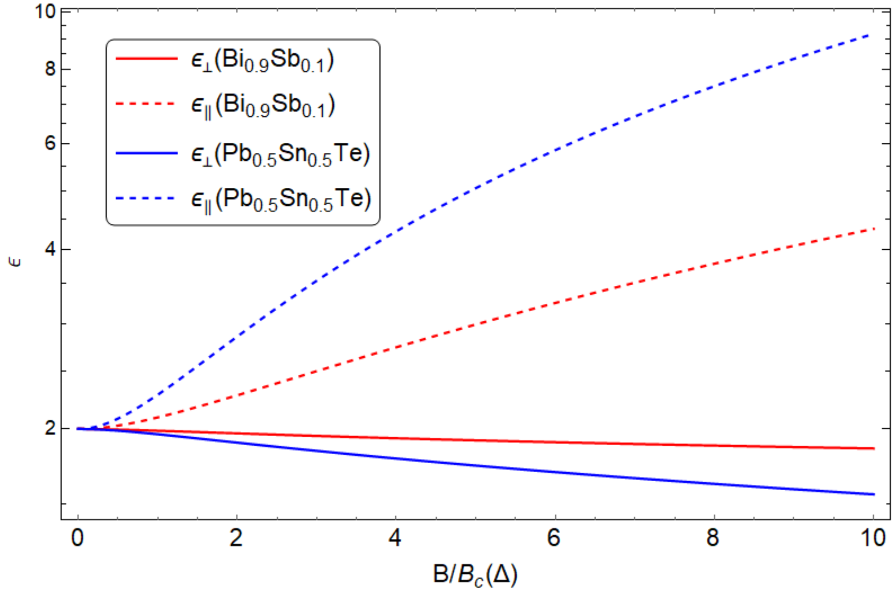
$$v_{ph} = \sqrt{1 + \frac{2\alpha_D(\omega^2 - 2eB)(n_e+n_p)}{4\alpha_D eB(n_e+n_p) + \pi((\omega^2-2eB)^2 - 4\omega^2)}}$$

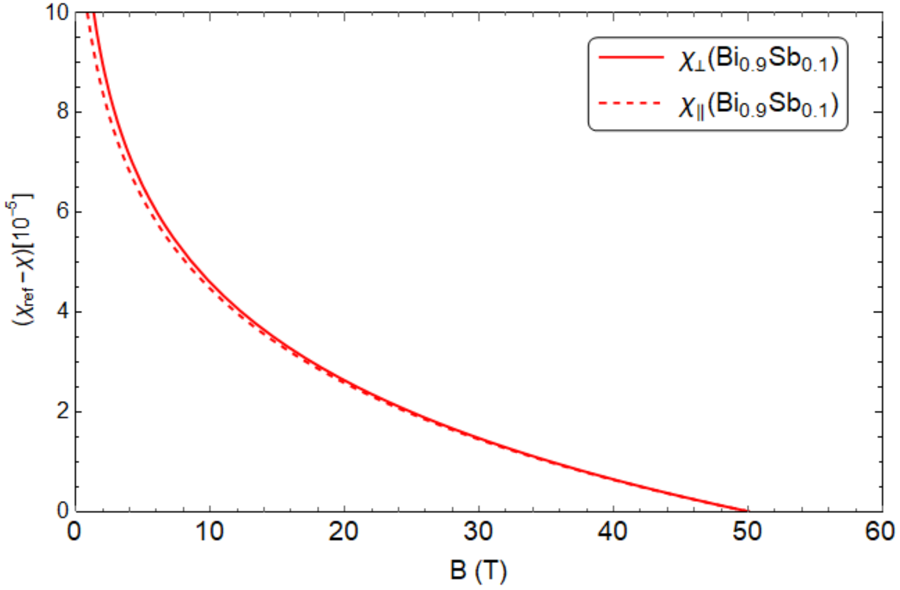
Solución analítica



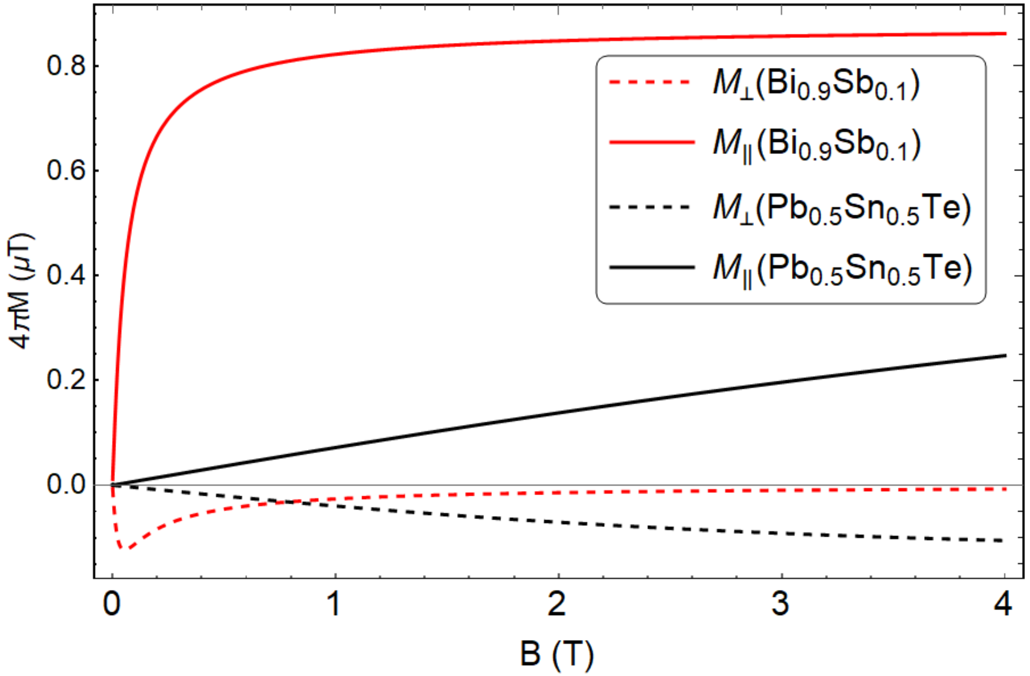
Propiedades de los materiales de Dirac

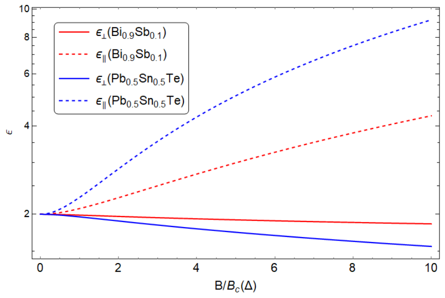


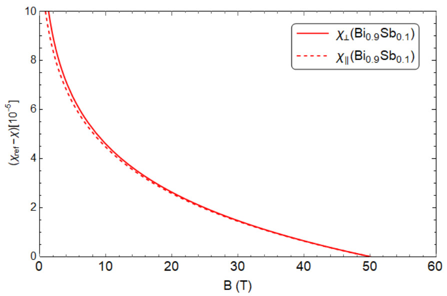


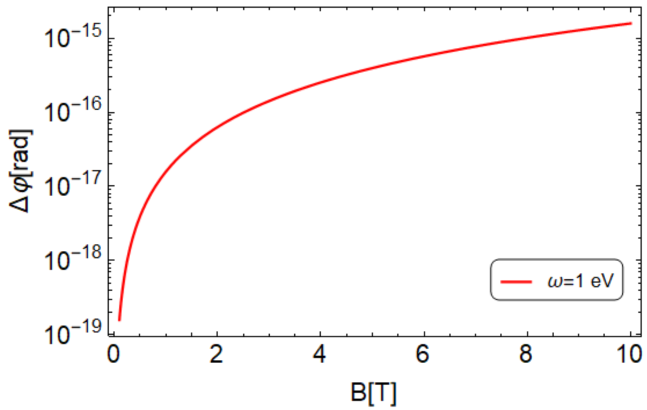


Propiedades de los materiales de Dirac







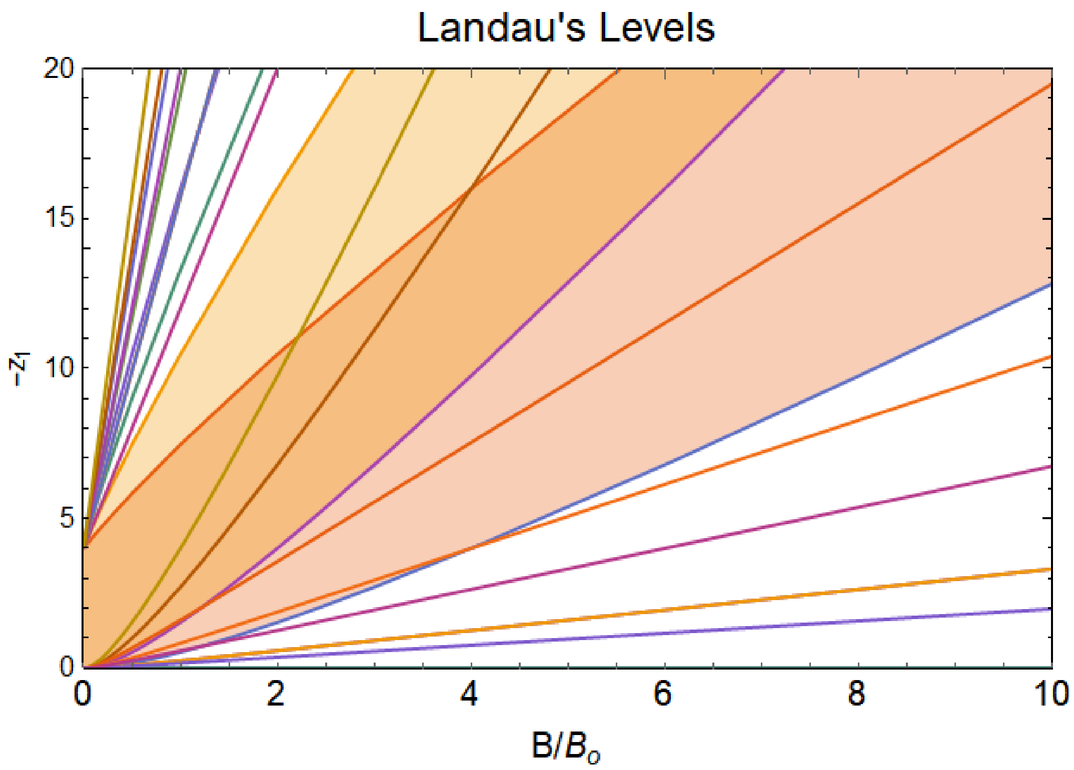


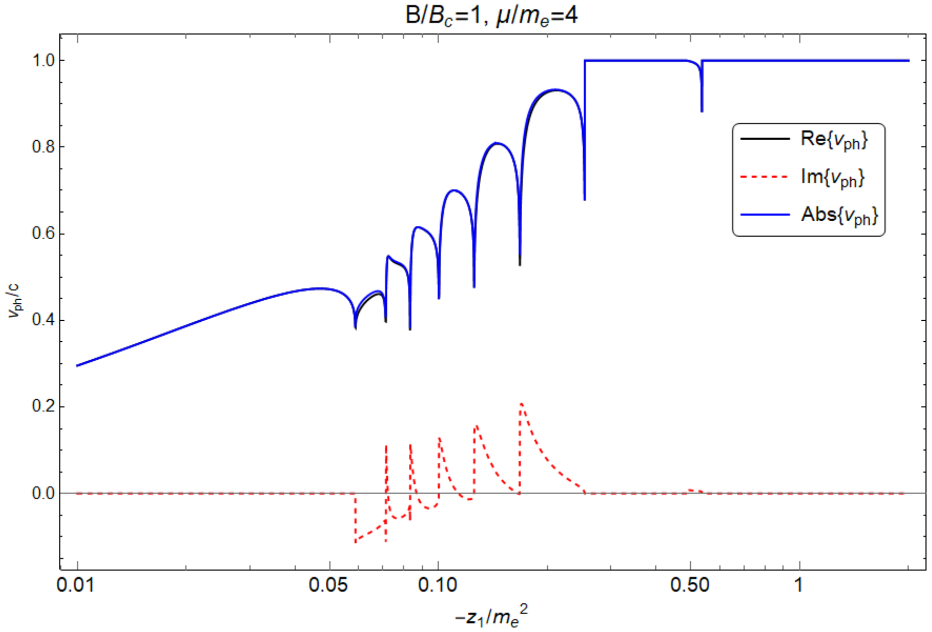
Comparar con las mediciones experimentales en busca de una corrección de vacío!

Metodología

Ley de Dispersión: Medio

- Límite Gas Degenerado $(T \ll \mu)$ $\quad n_L = \dfrac{\mu^2 - m_e^2}{2eB}$





Ley de Dispersión. Materiales de Dirac

Caso Grafeno 2D $(\Delta = 0)$

$$\kappa_M^{(2)} = -\frac{e^3 B}{\pi^2} \sum_{n_l=1} \frac{1}{\varepsilon} \left[\frac{F_n^{(2)}(k_\perp)}{4} \left(1 - \frac{(2eB(2n+1) - \omega^2)(2eB - \omega^2)}{4\omega^2 p_0^2} \right) - N_n^{(1)}(k_\perp) \left(\frac{2eB - \omega^2}{4\omega^2 p_0^2} \right) \right] (n_e + n_p)$$

$$\kappa_M^{(2)} = t = h - k_\perp^2 g$$

n=0

$$\lim_{\Delta \to 0} \lim_{n \to 0} g, h = 0$$

$$N_{n_l n_l'} \to 0 \qquad \varepsilon = \sqrt{2enB} \quad \Rightarrow \quad \sum_{n=1}^{\infty} \ldots$$

Ley de Dispersión. Materiales de Dirac

Caso Grafeno 2D $(\Delta = 0)$

n=0 $\lim_{\Delta\to 0}\lim_{n\to 0} g, h = 0$ $N_{n_l n_l'} \to 0$ $\varepsilon = \sqrt{2enB}$ ⇒ $\sum_{n=1}^{\infty} \dots$

Ley de Dispersión. Materiales de Dirac

Caso 2D, campo fuerte en el nivel de Landau más bajo ($n_l = 0$)

$$\varepsilon = \sqrt{\Delta^2 + p_3^2 v_F^2 + 2enB} \rightarrow \sqrt{\Delta^2 + p_3^2 v_F^2}, \quad F^{(2)}_{n_l n_l'} \rightarrow e^{-k_\perp^2/2eB} \approx 1 - k_\perp^2/2eB \quad N_{n_l n_l'} \rightarrow 0$$

$$\kappa^{(2)} = -\frac{e^3 B}{4\pi^2}\left(1 - \frac{k_\perp^2}{2eB}\right)\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon}\left(1 + \frac{(2eB - \omega^2)^2}{4\omega^2(p_3^2 - p_0^2)}\right)(n_e + n_p)$$

$$\kappa^{(2)} = -\frac{e^3 B}{4\pi^2}\left(1 - \frac{k_\perp^2}{2eB}\right)\frac{1}{\varepsilon}\left(1 - \frac{(2eB - \omega^2)^2}{4\omega^2 p_0^2}\right)(n_e + n_p)$$

Quitar np

$$k_\perp^2 - \omega^2 = \kappa^{(2)}$$

$$v_{ph} = \sqrt{1 + \frac{2\alpha_D(\omega^2 - 2eB)(n_e + n_p)}{4\alpha_D eB(n_e + n_p) + \pi((\omega^2 - 2eB)^2 - 4\omega^2)}}$$

Pregunta 1:

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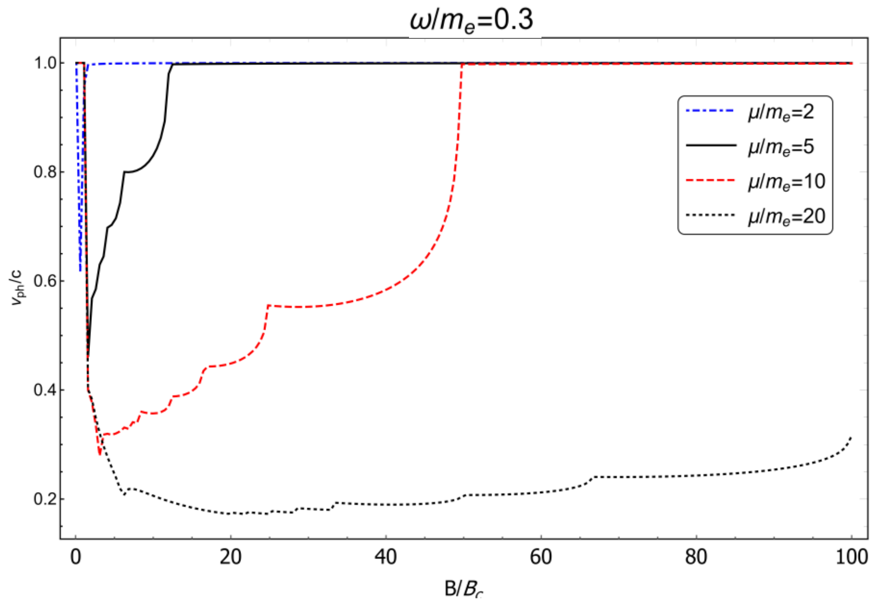


$$v_f = \frac{\omega}{k_\perp} \qquad v_f < 1$$

Información, velocidad de la fase

$$v_g = \frac{\partial \omega}{\partial k_\perp} = v_f + k_\perp \frac{\partial v_f}{\partial k_\perp}$$

Energía, velocidad de la envolvente

Vacío $\frac{dv_f(B)}{dk_\perp} = 0 \qquad v_f^V = v_g^V$

Medio $\frac{dv_f(k_\perp, \omega, B, T)}{dk_\perp} \neq 0 \qquad v_f^V \neq v_g^V$

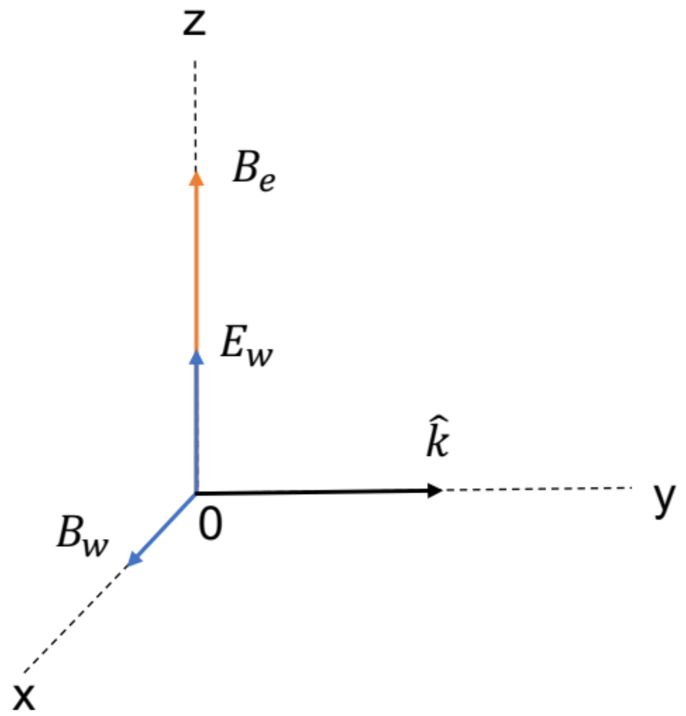
Brillouin (1946) “que en un medio, si también se produce absorción, la velocidad del grupo deja de tener un significado físico claro.”

$c = 299\,792{,}458\, m/s \qquad n = v_f^{-1}$

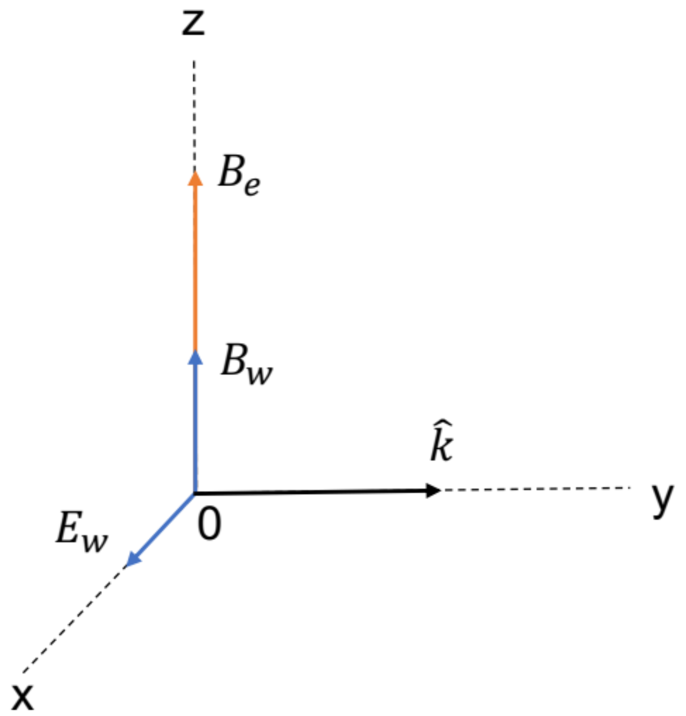
$$v_g = \left(\frac{\partial(Re\{k_\perp\})}{\partial \omega}\right)^{-1}$$

Dispersión anómala $v_g > 1, < 0, \rightarrow \infty$

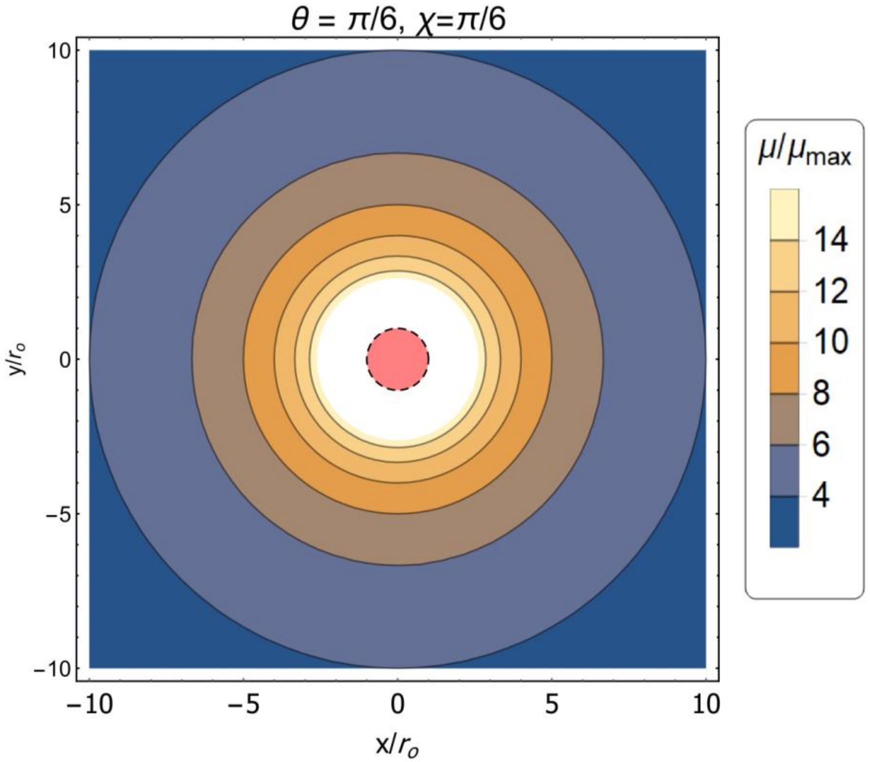
Mode 2

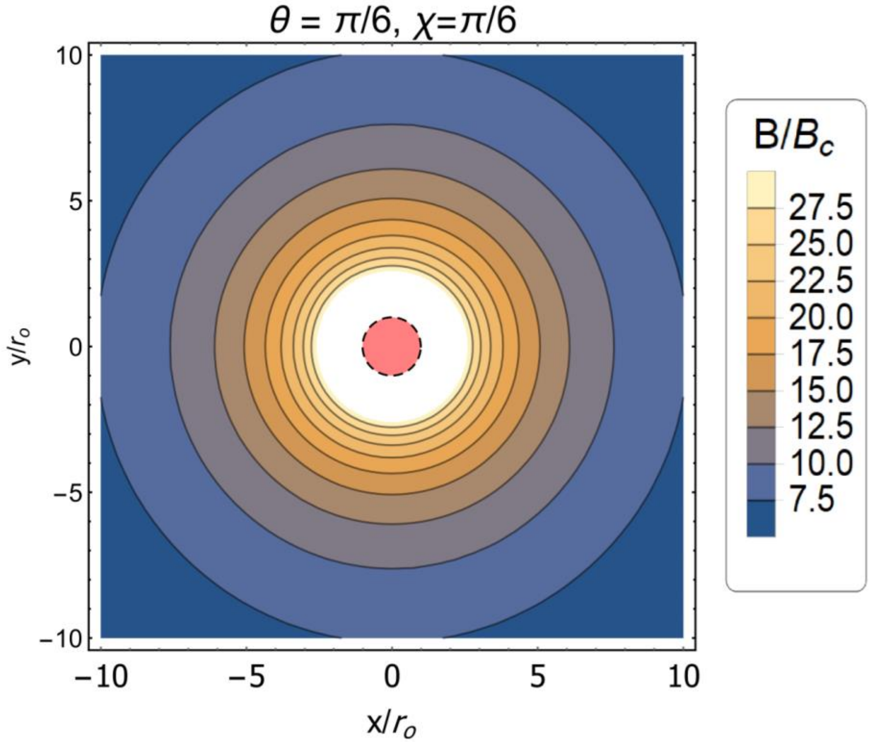


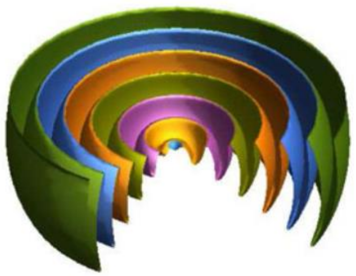
Mode 3

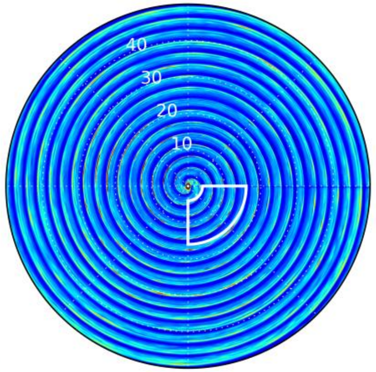


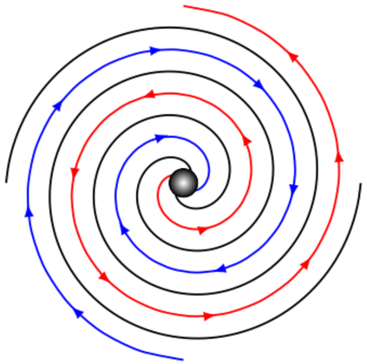
Aplicación Astrofísica: Estrellas de neutrones

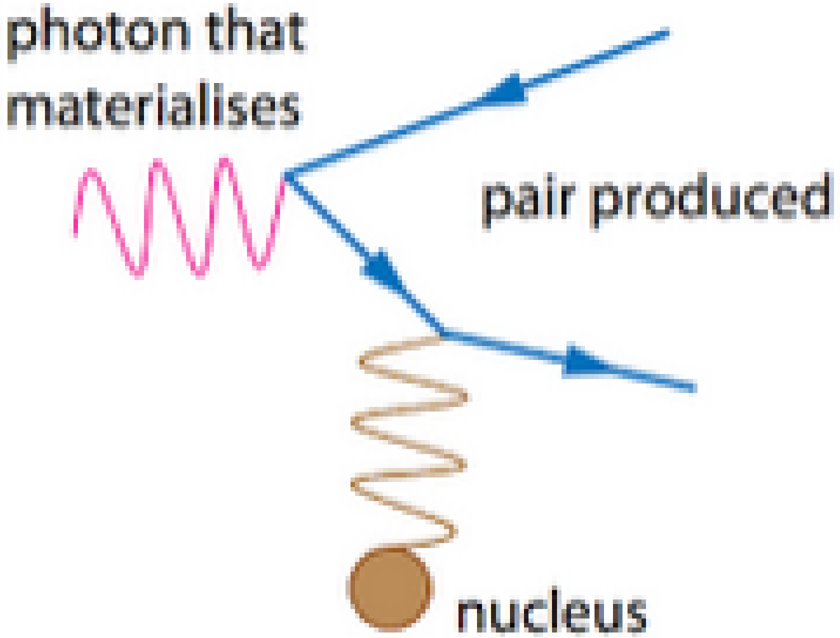












b) Compton scattering: